

# ESSAYS IN TIME SERIES ANALYSIS IN THE FREQUENCY DOMAIN

A Dissertation

Presented to the Faculty of the Graduate School  
of Cornell University

in Partial Fulfillment of the Requirements for the Degree of  
Doctor of Philosophy

by

Jingxian Zheng

May 2013

© 2013 Jingxian Zheng  
ALL RIGHTS RESERVED

# ESSAYS IN TIME SERIES ANALYSIS IN THE FREQUENCY DOMAIN

Jingxian Zheng, Ph.D.

Cornell University 2013

In financial markets, economic relations can change abruptly as the result of rapid market reactions to exogenous shocks, or, alternatively, change gradually over a long time span incorporating various activities and responses from multiple market participants at different points in time. Studies on financial contagion concentrate on such changes in interdependence relations among economies, industries, or institutions. These changes in interdependence can be measured by the instabilities in the covariance structure of two asset returns, which consists of the contemporary covariance and all lag orders of the cross-autocovariances. By Fourier Transform, a spectral density function contains equivalent information as covariance function. Therefore, any changes in the covariance structure can be captured by changes in the spectral density function.

In the first chapter, Detection of Abrupt Structural Changes: A Spectral Approach, I propose a spectrum-based estimator to detect abrupt changes in the covariance structures. In this approach, detecting these abrupt changes is equivalent to locating the step discontinuities in the time-varying spectral densities and cross-spectral density. The estimator can then be implemented based on a comparison of the left and right limit spectra of the potential time spot. This method brings together and improves upon two strands of the literature on structural changes. Compared to the existing estimators in the structural break literature which mainly consider structural changes as discrete level shifts in an observation period, my method is more general in allowing occasional breaks to

occur in a smooth change circumstance approximated by locally stationary processes, thus subsuming level shifts as a special case. My method also extends the literature that focuses on smooth changes approximated by local stationarity by relaxing the assumption of continuity and by introducing abrupt changes.

I empirically apply the estimator to pairs of index returns in the subprime mortgage, stock, and bond markets during the 2007 subprime crisis and the 2008 global financial crisis. The empirical results show that during the crises, abrupt changes are apt to be but not necessarily triggered by specific shocking events. Moreover, most of the changes in the dependence structures of index returns are closely related to the changes in the marginal covariance structures of the returns. However, not all of the changes in marginal covariance structures lead to changes in the cross-covariance structures.

The detection method is adopted in the second chapter, Post-Crisis Global Liquidity and Financial Spillover: From U.S. to Emerging Markets. This paper empirically investigates the linkages between U.S. markets and emerging markets to identify the global liquidity and financial spillover after the 2008 global financial crisis. A two-step method is adopted to capture dynamic patterns and structural changes in the linkages between the bond and stock markets in U.S. and BRICS. The results show that most abrupt changes in the U.S. and BRICS markets were due to specific shocking events in the U.S. markets and the abrupt changes were globally synchronous after the global financial crisis. Furthermore, there was a temporary liquidity spillover from U.S. to some of the emerging markets, as the U.S. Federal Reserve implemented the second round of quantitative easing. Overall, the lead effects of the U.S. bond and equity markets were much more significant than the spillover effect of the U.S. liquidity. Thus the financial spillover was more likely through the correlated-information

channel than the liquidity channel.

The third chapter, Generalized Spectral Estimation of Time Series Conditional Moment Restriction Models with Infinite Dimensional Conditioning Set, is coauthored with Zhaogang Song. We propose a generalized spectral estimator via frequency domain methods for a class of time series models. This class of time series models is defined by conditional moment restrictions with infinite dimensional conditioning set. The framework is general enough to cover most models which can be represented by conditional moment restrictions as special cases, including IV, nonlinear dynamic regression models, and rational expectations models such as consumption based asset pricing models(CCAPM). The estimator is obtained by minimizing the Cramr-Von Mises distance between the unrestricted generalized spectral distribution function and the model-implied correspondent, which is equivalent to setting a pairwise nonlinear dependence measure as close as possible to zero for each time lag order. It can be understood as a GMM estimator based on a set of moment conditions which grow with sample size. Not only is the infinite dimensional conditioning information is embedded in this estimator, but also the nonlinear dependence is captured. Another feature is the simplicity since its implementation does not require selecting any user-chosen number. Simulation studies show that unlike existing estimators which can only deal with either linear dependence or a fixed finite number of conditioning variables separately instead of simultaneously, our proposed estimator are free of any identification problem as expected by incorporating both nonlinear dependence and infinite dimensional conditioning information. An empirical application for estimating CCAPM is conducted and we find that economic agents are much more risk-averse according to our estimator than what Hansen and Singleton's(1982) GMM estimation results imply.

## **BIOGRAPHICAL SKETCH**

Jingxian Zheng was born in Xiamen, China in October, 1984. She studied Finance at Xiamen University in China and earned her B.A. degree in Economics in June, 2007. She then began her study at Cornell University in 2007 and was awarded the M.A. degree in Economics in August, 2011. She will receive her Ph.D. degree in May, 2013.

Dedicated to my parents, Ming and Yanping.

## ACKNOWLEDGEMENTS

This dissertation could not be finished without the guidance of my committee members and help and support from my friends and family.

First of all, I would like to express my deepest gratitude to my advisor, Professor Yongmiao Hong, for his caring, patience, and excellent guidance in research. I became interested in spectral analysis when I took the advance econometric course Nonlinear Time Series Analysis taught by Professor Hong. My first research idea in econometrics, though pretty preliminary, was inspired in this course. I thank Professor Hong for providing me with a good atmosphere for doing research. I was always encouraged by him in trials of new ideas, and sharing and exchanging ideas with my fellow students in the weekend study group organized by Professor Hong.

I also really appreciate the stimulating discussions and encouragement from my committee, Professor G. Andrew Karolyi and Professor Nicholas M. Kiefer. Without their incisive comments on my work, I would have spent much longer time on getting on board and keeping on the right track.

I owed thanks to all of my friends at Cornell, for supporting me on my study and helping me on my personal life. Last but not least, I thank my parents, for their support, protection and unconditional love.



## TABLE OF CONTENTS

Biographical Sketch . . . . .	iii
Dedication . . . . .	iv
Acknowledgements . . . . .	v
Table of Contents . . . . .	vi
List of Tables . . . . .	viii
List of Figures . . . . .	ix
<b>1 Introduction</b>	<b>1</b>
1.1 Detection of Abrupt Structural Changes: A Spectral Approach . .	1
1.2 Post-Crisis Global Liquidity and Financial Spillover: From U.S. to Emerging Markets . . . . .	7
1.3 Generalized Spectral Estimation of Time Series Conditional Moment Restrictions Models with Infinite Dimensional Conditioning Set . . . . .	12
<b>2 Detection of Abrupt Structural Changes: A Spectral Approach</b>	<b>20</b>
2.1 Literature review . . . . .	20
2.2 Covariance structural changes versus parameter instability in regression models . . . . .	22
2.3 Abrupt structural changes and local stationarity . . . . .	25
2.4 Estimation for limit spectra . . . . .	27
2.5 Test statistic and its asymptotic distribution . . . . .	28
2.6 Estimation for abrupt changes . . . . .	33
2.6.1 Estimation for A Single Change . . . . .	33
2.6.2 Estimation for multiple abrupt changes . . . . .	36
2.7 Finite-sample performance . . . . .	39
2.8 Empirical application . . . . .	45
2.9 Conclusion . . . . .	56
<b>3 Post-Crisis Global Liquidity and Financial Spillover: From U.S. to Emerging Markets</b>	<b>58</b>
3.1 Funding Liquidity and Market Liquidity . . . . .	58
3.2 Data . . . . .	60
3.2.1 Measure of funding liquidity . . . . .	61
3.2.2 Financial Variables in Bond and Equity Markets . . . . .	64
3.3 Methodology . . . . .	66
3.3.1 Detection of abrupt changes . . . . .	67
3.3.2 Time-varying Vector Autoregression . . . . .	68
3.4 Empirical Results . . . . .	69
3.4.1 U.S. versus Russia . . . . .	69
3.4.2 U.S. versus Brazil . . . . .	75
3.4.3 U.S. versus India . . . . .	78

3.4.4	U.S. versus China . . . . .	82
3.4.5	U.S. versus South Africa . . . . .	85
3.5	Conclusion . . . . .	90
<b>4</b>	<b>Generalized Spectral Estimation of Time Series Conditional Moment Restrictions Models with Infinite Dimensional Conditioning Set</b>	<b>91</b>
4.1	The Generalized Spectral Estimator . . . . .	91
4.2	Asymptotic theory . . . . .	100
4.3	Simulation Studies . . . . .	104
4.3.1	Simulation Design . . . . .	104
4.3.2	Simulation Evidence . . . . .	106
4.4	Empirical Application: Estimating the CCAPM . . . . .	112
4.5	Conclusion . . . . .	118
<b>A</b>	<b>Appendix of Chapter 2</b>	<b>120</b>
A.1	Figures and Tables . . . . .	120
A.2	Mathematical Proofs . . . . .	136
<b>B</b>	<b>Appendix of Chapter 4</b>	<b>143</b>
	<b>Bibliography</b>	<b>145</b>

## LIST OF TABLES

2.1	Simulation Results of $\widehat{\lambda}$ at 5% . . . . .	40
2.2	Abrupt Changes in the Cross-covariance Structures Between the Subprime and the Security Markets . . . . .	47
2.3	Abrupt Changes in the Marginal Covariance Structures . . . . .	48
2.4	Abrupt Changes in the Cross-covariance Structures in the Subprime Market . . . . .	51
2.5	Abrupt Changes in the Cross-covariance Structures in the Security Markets . . . . .	53
3.1	Abrupt Changes Detected among the financial variables in Russia-U.S. . . . .	70
3.2	VAR(P) Coefficients for Russia-U.S. . . . .	73
3.3	Abrupt Changes Detected between the U.S. -Brazil variables . . . . .	75
3.4	VAR(P) Coefficients for Brazil-U.S. . . . .	77
3.5	Abrupt Changes Detected among the financial variables in India-U.S. . . . .	80
3.6	VAR(P) Coefficients for India-U.S. . . . .	81
3.7	Abrupt Changes Detected Among the Financial Variables in China-U.S. . . . .	83
3.8	VAR(P) Coefficients for China-U.S. . . . .	84
3.9	Abrupt Changes Detected among the Financial Variables in South Africa-U.S. . . . .	86
3.10	VAR(P) Coefficients for South Africa-U.S. . . . .	88
4.1	GSE vs DL for Example-1 . . . . .	107
4.2	GSE vs PSE for Example-2 . . . . .	110
4.3	GSE vs DL for Example-3 . . . . .	111
4.4	GMM, PSE and GSE for CCAPM . . . . .	118
A.1	Possible Influential Events . . . . .	135

## LIST OF FIGURES

2.1	DGP 0: Frequency Distribution of $\widehat{\lambda}$ . . . . .	41
2.2	DGP 1: Frequency Distribution of $\widehat{\lambda}$ . . . . .	42
2.3	DGP 2: Frequency Distribution of $\lambda$ . . . . .	42
2.4	DGP 3: Frequency Distribution of $\widehat{\lambda}$ . . . . .	43
2.5	DGP 4: Frequency Distribution of $\widehat{\lambda}$ . . . . .	44
3.1	U.S. TED Spread . . . . .	61
3.2	TED Spreads for Emerging Markets . . . . .	62
3.3	Bond Indexes for U.S. and Emerging Economies . . . . .	65
3.4	Stock Indexes for U.S. and Emerging Economies . . . . .	65
4.1	Objective Functions of GSE and DL for Example 1 . . . . .	108
4.2	Scatters of GSE and DL for Example-1 . . . . .	109
4.3	Objective Functions of GSE and PSE for Example-2 . . . . .	111
4.4	Objective Functions of GSE and DL for Example-3 . . . . .	112
A.1	Detected Abrupt Changes Between ABX_AAA and Corporate Bonds . . . . .	120
A.2	Detected Abrupt Changes Between ABX_AAA and Treasury Bonds	121
A.3	Detected Abrupt Changes Between ABX_AAA and S&P 500 . . . .	122
A.4	Detected Abrupt Changes Between ABX_AA and Corporate Bonds	123
A.5	Detected Abrupt Changes Between ABX_AA and Treasury Bonds	124
A.6	Detected Abrupt Changes Between ABX_AA and S&P 500 . . . .	125
A.7	Detected Abrupt Changes Between ABX_A and Corporate Bonds	126
A.8	Detected Abrupt Changes Between ABX_A and Treasury Bonds .	127
A.9	Detected Abrupt Changes Between ABX_A and S&P 500 . . . . .	128
A.10	Detected Abrupt Changes Between ABX_BBB and Corporate Bonds . . . . .	129
A.11	Detected Abrupt Changes Between ABX_BBB and Treasury Bonds	130
A.12	Detected Abrupt Changes Between ABX_BBB and S&P 500 . . . .	131
A.13	Detected Abrupt Changes Between ABX_BBB- and Corporate Bonds . . . . .	132
A.14	Detected Abrupt Changes Between ABX_BBB- and Treasury Bonds	133
A.15	Detected Abrupt Changes Between ABX_BBB- and S&P 500 . . . .	134

## CHAPTER 1

### INTRODUCTION

#### **1.1 Detection of Abrupt Structural Changes: A Spectral Approach**

The structural changes in economic relations have long been of interest in the time series analysis. For economists, structural changes refer to the long-term widespread changes in the underlying economic structure. These changes are due to various factors such as policy and institutional decisions, changes in resources, and the behavioral patterns of individuals. For econometricians or statisticians, structural changes mean changes in parameters or statistical characteristics. These two definitions are coordinated in the sense that economic relations are specified by parameters or statistical characteristics in econometric models. Extensive work in the finance literature has verified the prevalence of structural instability in economic relations. Empirically, the means, volatilities, autocovariances, and cross-covariances of asset returns change across time. For example, Lo and Mackinlay (1990) first document the lead-lag effect between large and small stock returns. Hou (2007) shows that the cross-autocorrelations between the returns of large and small firms vary in different time samples.

The economic relations in financial markets are apt to change abruptly as a result of rapid market reactions to exogenous shocks, such as crises. They can also change gradually over a long time span incorporating various activities and responses from multiple market participants at different points in time. One of the related topics that has attracted a lot of attention is the study on financial

contagion that concentrates on such changes in dependence structures across economies, industries, or institutions. Among others, Bae, Karolyi, and Stulz (2003) propose an approach to measure of the financial contagion defined as the coincidence of extreme return shocks within a region and across regions.<sup>1</sup> Such changes in dependence structures can be sudden or gradual, without observable starting and end points. For example, during a crisis, when financial institutions suddenly stop or slow down lending activity, the interconnectedness among institutes is believed to change abruptly.<sup>2</sup> In contrast, rebuilding the financial architecture is a much more gradual process as it involves various activities and responses from multiple parties, such as bankruptcies and acquisitions, fiscal stimulus, monetary policy expansion, and institutional bailouts. In essence, the timing and the dynamic pattern of structural changes can be too ambiguous to be determined by specific events, making the detection of such changes rather difficult.<sup>3</sup> This paper provides a formal procedure to detect abrupt changes in an instable dependence structure that can also contain smooth changes. To the best of my knowledge, this paper is the first to distinguish abrupt changes from

---

<sup>1</sup>There is no consensus on the exact definition of the financial contagion. Longin and Solnik (1995, 2003) investigate conditional correlation and extreme correlation in the international equity markets, respectively. Forbes and Rigobon (2001, 2002) define the financial contagion as a significant change in cross-country/market correlations following a crisis in one or more countries/markets. Bekaert, Harvey, and Ng (2005) use a two-factor model and capture the contagion by time-varying betas. Longstaff (2010) investigates the mechanism in which the contagion is propagated from the subprime market to the security markets by detecting changes in linkages across markets. Among many others, Billio, Getmansky, Lo, and Pelizzon (2012) detect the propagation of the contagions among banks, insurance, brokers, and hedge funds during the 2008 global financial crisis in a similar way.

<sup>2</sup>Brunnermeier (2009) investigates liquidity and the credit crunch as the immediate result of the subprime crisis.

<sup>3</sup>For example, subprime mortgage became a watch-word in public after February 7, 2007 when HSBC blamed soured U.S. subprime loans for its first-ever profit warning, and then landmark problems began when New Century Financial filed for bankruptcy on April 2. A sequence of negative events thereafter accelerated the crisis and spread the crisis from the subprime market to the entire financial sector. However, the subprime mortgage market structure may already have started to change since mid-2006 when the property bubble began to unwind. Thus, it is extremely difficult, if not impossible, to detect any structural change by the timeline of specific events.

smooth changes in an instable dependence structure.

To investigate the changes in the dependence relations across markets, this paper, like most of the relevant literature, specifically focuses on the coefficients in regression models or covariance-related statistical characteristics. When a structure changes fast in a short period of time, the change can be justified as an abrupt change; otherwise, when the change is gradual and slow, it can be justified as a smooth change. On the one hand, an abrupt change, also introduced as a structural break in some literature, is a discrete level shift of the parameters or statistical characteristics during the observation period. On the other hand, structural changes can be assumed to be smooth over time. Because of the stylized fact that financial markets often change their behaviors abruptly at the start of a financial crisis and persist or evolve gradually to the changed behavior, the discrimination between abrupt and smooth changes is important in identifying the timing of a crisis and capturing the dynamic patterns of the dependence across markets.

In conventional methods, the investigation of the changes in linkages consists of two steps. First, the data sample is split by calendar year or by some specific distress events. Then the time-invariant coefficients in the regression models or correlation parameters are estimated using the data in each subsample. The implicit consideration is that the abrupt changes occur at the end of years or right after certain events, and that the changed relations are persistent until the next abrupt change. Though pervasively applied, this kind of method is problematic. First, the coincidences of abrupt changes and calendar year-ends or certain events are not justified, as the example of the subprime crisis shows. Some events that appear impactful can cause merely transient distortions in the

data rather than permanent structural changes. Second, simple splitting methods can mislocate the real breaks and therefore render the statistical inferences and predictions unreliable. Cutting data at a time spot that actually undergoes a smooth change or does not experience structural changes at all can result in inefficient use of information. Furthermore, the conventional models do not allow the dependence structures to change smoothly.

In contrast, the changes in linkages can be directly measured by the instabilities in the cross-covariance structure of two asset returns. A cross-covariance structure includes not only the cross-sectional co-movement but also all of the orders of cross-autocovariances. Therefore, a change in any order of the cross-autocovariances is considered as a change in linkage. In this paper, I propose a spectrum-based method to detect abrupt changes in the dependence structures. The spectrum-based method is more natural and convenient than the time domain method such as the regression models for testing and estimating covariance structures, since a spectrum is the Fourier transform of covariances up to all possible lags thus being considered as a weighted average over all possible covariances. For a time domain method, e.g., in a regression model such as a vector autoregression (VAR), the lags have to be pre-determined by the Akaike Information Criterion (AIC) or other similar criteria. This is computationally burdensome and perilous when the time series undergoes some structural changes. The advantage of the spectrum-based method is that in the frequency domain, when processes are covariance-stationary, the spectrum is time-invariant. When stochastic processes are nonstationary with covariance structural changes, the joint distributions of the time series change over time. Consequently, parameters such as means and covariances of certain lags evolve over time. Meanwhile, as a summation function of covariances for all possible



lags, the spectral density changes over time. If a covariance structure changes smoothly, then the spectrum changes smoothly as defined by Priestley (1965) and Dahlhaus (1996). Therefore, by using estimates that involve only local functions of the data, the time-varying spectrum can be estimated in terms of the average spectrum of the process in the neighborhood of any particular time instant. This extensive spectrum denoted as an evolutionary spectrum or a local stationary spectrum provides a convenient way for interpreting the results of a conventional spectral analysis applied to data from nonstationary processes. Furthermore, the spectral densities are continuous functions in accordance with the smoothness of the changes, and when processes are stationary, the evolutionary spectrum reduces to the conventional one. To this end, in the frequency domain, measuring the smooth structural changes has been translated into measuring the smooth variation of the spectrum throughout the time period. In this paper, I introduce break points into the continuous time-varying spectra, where the break points are defined as the step discontinuous points in the spectral density functions. Therefore, only one side rather than the whole neighborhood of observations is used to estimate the limit spectrum at a time instant if it is a break point.

The spectral density function is step discontinuous at each changing point in time and continuous elsewhere. Thus, the estimation of the abrupt change points is constructed based on a one-sided kernel estimator for locally stationary processes. For the case of a single break, the data in the left neighborhood of each potential time instant is used to compute the estimate of the left limit, and the symmetric procedure is taken to compute the estimate of the right limit. Then the largest difference in the left and right limit estimates among all time instants

is defined as the estimate of the true changing point, given a certain criterion.<sup>4</sup> This method is extended to a multiple change case in which the locations and the number of abrupt changes can be unknown. This is more realistic for most empirical applications. In essence, I split the data into segments with an equal number of observations, such that each segment contains at most one abrupt change. Then all abrupt changes are tested and estimated with the same power and the same size. In each segment, abrupt changes are detected in the similar way as in the single change case.

Simulation results show that the estimator performs well in finite samples for data generating processes with either abrupt or smooth changes and processes with both changes. Compared with estimation methodologies in the existing literature on structural changes, the method proposed in this paper has the following appealing features. First, the method connects the literature with the two justifications of structural changes (i.e., abrupt or smooth) by providing a way to accommodate both abrupt and smooth structural changes and to distinguish the abrupt changes from a smooth changing structure or a constant structure. Second, the proposed one-sided kernel estimator stays consistent for entire processes, regardless of whether a process keeps stationary or changes abruptly or smoothly. Furthermore, this estimator enables testing and estimation based on the evolutionary spectrum to detect and measure the abrupt changes in the linear dependent structure. The proposed spectral tool is natural and convenient for measuring the dependent structure since the spectrum and the covariance function contain equivalent information. It is capable of distinguishing abrupt changes from smooth changes explicitly, which generates interesting empirical implications, and it is easily extendable to multivariate econo-

---

<sup>4</sup>In this paper, I focus on a Kolmogorov-Smirnov type criterion (as detailed in Section 5 in Chapter 2).

metric models, such as time-varying VAR and dynamic factor models allowing nonstationary time series.

I apply the estimator to index returns data to empirically detect abrupt changes in the interdependence relations across subprime mortgage, stock, and bond markets in the U.S. during the 2007 subprime crisis and the 2008 global financial crisis. The empirical results show that the covariance structure does have abrupt changes right after some bad events, such as the sequence of subprime-related bad news in mid-September 2008, including the nationalization of Fannie Mae and Freddie Mac, the takeover of Merrill Lynch by Bank of America, the bankruptcy of Lehman Brothers, and the consequent downgrades on AIG's credit rating. On the other hand, abrupt changes are not necessarily triggered by specific events. For example, the abrupt change detected in December 2008 can be considered as an accumulative result of the U.S. housing bubble burst and the global financial crisis. What's more, most of the changes in the dependence structures are closely related to the changes in the marginal covariance structures of the returns, but not all changes in the marginal covariance structures lead to changes in the cross-covariance structures.

## **1.2 Post-Crisis Global Liquidity and Financial Spillover: From U.S. to Emerging Markets**

Global liquidity and financial spillover from the U.S. to emerging economies have attracted a lot attention since the 2007 U.S. subprime mortgage crisis. As the crisis spread and soon became the catalyst for the global financial turmoil, the discussions mainly focused on the global liquidity crunch, illiquidity

spillover, and financial contagion. Most recently, as a result of a series of unprecedented unconventional policy interventions by advanced economies, the possible liquidity spillover to emerging markets attracted more attention. In this paper, without exogenously distinguishing these two phrases, I use an innovative econometric approach to investigate the existence and the dynamic patterns of the liquidity spillover.

Funding liquidity is the ease with which traders can obtain funding from financiers. Discussed by Brunnermeier (2009), during 2007 and 2008, the joint reinforcements of four mechanisms evaporated the funding liquidity and amplified the relatively modest losses in the mortgage market into a full-blown financial crisis. As the U.S. housing bubble burst and asset prices dropped, financial institutions' capitals eroded, while lending standards and margins tightened. Both effects led to fire-sales, and further pulled down asset prices and tightened margins. This mechanism is the "liquidity spirals" caused by borrowers' balance sheet effect. The second mechanism is the drying-out of lending channel when financiers started to hoard funds. Thirdly, the runs on financial institutions caused a sudden erosion of bank capital. Last but not least, financial institutions had to hold additional funds to avoid counterpart credit risk.

The severe financial turmoil triggered the U.S. Federal Reserve to launch a series of unconventional policy measures, after cutting the key interest rate to close to the zero lower bound. In late 2007 and early 2008, the U.S. Federal Reserve implemented several programs associated with direct lending to financial institutions, to address the extremely limited availability of credit in short-term funding markets. As the financial turmoil intensified dramatically, after the collapse Lehman Brothers and a series of bad news in financial mar-

kets, the Federal Reserve launched the first round of quantitative easing (QE1) on November 26th 2008 and ended it on March 1st 2010. The measures aimed at repairing the functioning of financial markets and mainly focused on large-scale asset purchases (LSAP) of mortgage-backed securities (MBS), debt obligations of government agencies, and longer-term Treasury securities. The second round of quantitative easing (QE2) implemented between November 3rd 2010 and June 30th 2011, primarily concentrated on purchases of Treasury securities. The aim of the policy measures were stimulating the U.S. economy by lowering yields, and pushing up asset prices in riskier markets. On September 21st 2011, the Federal Reserve launched “Operation Twist” and ended it in June 2012, to extend the maturity of securities held on its balance sheet by selling short-term Treasuries in exchange for the same amount of longer-term Treasury securities. The third round of quantitative easing was launched on September 13th 2012, focusing on the purchase of MBS.

The non-standard measures have raised debates for the effects on domestic markets<sup>5</sup>. They are also criticized by foreign policy-makers, especially the central banks of emerging economies, for having created excessive global liquidity, and thus caused the massive acceleration of capital flows to emerging markets since 2009. As a result, the capital inflow is widely blamed for appreciation pressures on currencies of emerging economies, a building-up of financial imbalances and asset price bubbles, high credit growth and a threat of economic overheating.

To capture the changes in the linkages among financial markets between U.S. and emerging economies and to identify the liquidity spillover effects, I

---

<sup>5</sup>For example, after the announcement of QE3, the riskier asset markets did not respond favorably as it had after previous QE installments.

choose bond and stock markets in BRICS (Brazil, Russia, India, China, and South Africa) as the representatives of emerging markets. The pattern of the spillover from U.S. to any of the economy is different from each other, which is associated with the international capital flow control and foreign exchange policies, maturity of security markets, and monetary and fiscal policies in the emerging economy. To measure liquidity in BRICS, I construct analogues of U.S. TED spread as the difference between the interbank offered rate and the Treasury bill rate for each economy. Moreover, the U.S.-Dollar denominated, investable by global investors bond and stock indexes are chosen as the benchmark indexes for the bond and stock markets in each emerging economy, respectively. The main features of taking these indexes are as follows: First, the indexes focus on the assets investable by foreign investors and are denominated in the U.S. Dollar, thus they reflect the capital inflow and include the cost of exchange rate changes; second, compared to the corresponding major domestic indexes, their dynamic patterns are similar, hence the selected indexes can precisely capture the general market conditions; third, these indexes synchronize with the U.S. markets, and thus the lead-lag effect caused by time difference can be averted.

In addition, I would also like to shed some light on the mechanisms of the financial spillover from advanced economies to emerging markets. In theoretical finance literature, the spillover mechanisms can be categorized into three channels. The first one is the correlated-information channel, in which a shock in a more-liquid market or a market with more rapid price-discovery can spill over to other markets via economic news directly or indirectly relevant for security prices in the other markets. The literature in this channel includes Dornbusch, Park, and Claessens (2000), Kiyotaki and Moore (2002), King and Wadhwani

(1990) and others. The second is the liquidity channel, or the portfolio balance channel. Through this channel, a shock to one financial market results in a rapid change in the overall liquidity of all financial markets. The models in Allen and Gale (2000), Kodres and Prisker (2002), Brunnermeier and Pedersen (2009) belong to this category. The third channel is termed as the risk-premium channel. By this mechanism, financial shocks in one market may affect the willingness of investors to bear risk in any market, and thus change the equilibrium risk premium. Vayanos (2004), and Acharya and Pedersen (2005) among others describe this mechanism. By the results estimate using a two-step method introduced in the following section, to some degree, I can identify the channel of the financial spillover.

The empirical results show that most of the abrupt changes in the linkage of U.S. and the emerging economies were triggered by specific negative shocking events or as cumulative result of series of negative shocking events in the U.S. market. Moreover, the U.S. liquidity spilled over to Russia, Brazil, and South Africa temporarily in 2010 and early 2011, as the U.S. Federal Reserve implemented the first two rounds of quantitative easing. Then the spillover effects disappeared, probably affected by the fear of investors about the contagion of European sovereign debt crisis and the anxiety of U.S. debt crisis. Compared to the spillover effects of U.S. liquidity, the lead effects of the U.S. stock and bond markets were more significant. For most of emerging economies, the linkages with U.S. reduced in some period after the global financial crisis, and returned to previous levels soon afterwards, which could be the result of policy responses of different economies.

In sum, the U.S. economic news affected the global markets and led the syn-

chronous structural changes. Furthermore, the lead effects of the U.S. bond and equity markets are much more significant than the spillover effects of liquidity. Therefore, the financial spillovers from U.S. to emerging markets were more likely through the correlated information channel than the liquidity channel.

### **1.3 Generalized Spectral Estimation of Time Series Conditional Moment Restrictions Models with Infinite Dimensional Conditioning Set**

In this article, we develop a methodology for estimating a class of time series models defined by conditional moment restrictions with infinite dimensional conditioning set. That is, the models establish that some parametric functions have zero conditional mean when evaluated at the true parameter value. Many econometric models are formulated this way in different areas of econometrics such as panel data, discrete choice, macroeconometrics and financial econometrics. But there is a notable specialty of models we consider here: the conditional information set is infinite dimensional instead of containing only a fixed and finite number of conditioning variables. In other words, the model can be defined as  $E[\rho(Z_t; \theta_0) \mid \mathcal{F}_{t-1}] = 0$ , where  $Z_t$  is a time series vector,  $\theta_0$  is the true parameters,  $\rho(\cdot; \theta_0)$  a known function, and  $\mathcal{F}_{t-1}$ , representing the conditional information set, contains an infinite number of conditioning variables, e.g., all the lagged variables of a time series,  $\{X_{t-1}, X_{t-2}, \dots\}$ . This class of models is actually very standard in both macroeconomics and finance literature. The family of the rational expectations models such as CCAPM defined by the Euler equation is a standard example (Hansen and Singleton, 1982; Singleton, 2006, Section 2.3.2)



for which  $\rho(\cdot; \theta_0)$  is the return scaled by the parametric marginal rate of substitution or stochastic discount factor.

Although there are many different estimators for models defined by conditional moment restrictions, most of them assume a finite dimensional conditioning set, i.e., only a fixed and finite number of conditioning variables show up. For models of cross section data whose conditioning set usually contains only a finite number of exogenous variables, these estimators pose no problems and will be consistent (Amemiya, 1974, 1977; Hansen, 1982; Newey, 1990, 1993; Robinson, 1987, 1991). However, for the time series models with an infinite dimensional conditioning set, most existing estimators, which only take a finite number of conditioning variables and cut the rest (Domínguez and Lobato, 2004), may not be consistent because all the lagged variables from the conditional information set should be included for the identification but only a limited number of them are taken into account. Furthermore, Hansen (2007), in a survey paper, actually argues that by allowing for moment conditions via using functions of variables in the conditioning information set, the asymptotic efficiency bound for GMM is improved. Similarly, our method focused on employing the infinite dimensional conditioning information can actually improve the efficiency bound substantially although our estimator is not guaranteed to achieve the bound.

The motivation of incorporating the infinite dimensional conditional information set is fairly evident in the empirical asset pricing literature which is focused on the persistence properties of the stochastic discount factors since they are "key determinants of the prices of long-lived securities" (Alvarez and Jermann, 2005). For example, several studies have shown that the stochastic

discount factors can be decomposed into two components, transitory components and permanent components(Alvarez and Jermann, 2005; Beveridge and Nelson, 1981)<sup>6</sup>. The presence of such permanent components, in terms of the models we consider here, tells us that all the conditional information in  $\mathcal{F}_{t-1}$  can help identify the models even if it is far away from time- $t$ . In fact, Alvarez and Jermann(2005) find that those permanent components must be large enough to be consistent with the low returns of long-term bonds relative to equity. Other recent works have also emphasized the need for understanding these low frequency components of stochastic discount factors which are important for the pricing of long-lived securities like stocks(Bansal and Yaron, 2004; Hansen, Heaton and Li, 2004).

Of course, if the processes involved in the models are all Markovian, the infinite dimensional condition information set  $\mathcal{F}_{t-1}$  will degenerate to a finite dimensional set consisting of just the previous lagged variable  $X_{t-1}$ . However, many recent studies have found that Markov property may not be a well-supported assumption for asset prices. For example, in the market microstructure literature, Easley and O'hara (1987) develop a structural model of the effect of asymmetric information on the price-trade size relationship. They show that trade size introduces an adverse selection problem to security trading and hence market makers pricing strategies must also depend on trade size. In consequence, the entire sequence of past trades is informative of the likelihood of an information event and prices typically will not follow a Markov process. Moreover, based on rigorous econometric procedures, Amaro de Matos and Fernandes (2007) find several asset prices exhibit non-Markovian property while

---

<sup>6</sup>Alvarez and Jermann's(2005) decomposition is multiplicative with the permanent component as a martingale while Beveridge and Nelson's(1981) decomposition is additive with permanent component following a random walk.

Chen and Hong(2008) find strong evidence against Markov assumption for time series of S&P500 stock index, 7-day Eurodollar interest rate and Japanese Yen exchange rate. To estimate time series models defined by conditional moment restrictions for these non-Markovian processes, especially when there is no prior information about which variables in the conditioning set are redundant, incorporating the infinite dimensional information is a natural choice.

Berkowitz(2001) does have an estimator making full use of the infinite dimensional information set by the power spectrum in the frequency domain. However, the characterization through power spectrum, which is the Fourier transform of the auto-covariance function, is only equivalent to the unconditional moment condition implied as a necessary condition of the conditional restriction. It is well known that the conditional moment restriction implies an infinite number of unconditional ones and there is a big gap between them. Consequently, the uniquely identified parameter in conditional moment models may not be identified by the unconditional restrictions. The latter may hold for several parameter values when the former just holds for a single value. Therefore, estimators based on the unconditional moment restrictions, which are not equivalent to conditional ones, are possibly inconsistent; see Domínguez and Lobato(2004) for details and some examples about this "conditional identification problem" due to the gap between unconditional and conditional moments.

The estimator we will propose in this paper is both free of the conditional identification problem and making full use of the infinite dimensional conditional information set. It is obtained by minimizing the Cramér-Von Mises distance between the unrestricted generalized spectral distribution function and the corresponding model-implied one, which is equivalent to setting a pairwise

nonlinear dependence measure as close as possible to zero for each time lag order. It can be understood as a GMM estimator based on a set of moment conditions which grow with sample size. Similar to Berkowitz(2001), this frequency domain approach makes the estimator contain all the lag orders of the conditioning variables, hence utilizing the infinite dimensional conditioning set completely. But unlike Berkowitz(2001), our estimator is based on a conditional mean dependence which can be viewed as a nonlinear generalization of the standard auto- covariance function. The resulted generalized spectral distribution function has the capability of capturing nonlinear dependence and hence our estimator is free of the conditional identification problem.

Of course, by focusing on efficient utilization of the infinite dimensional conditional information for identification, we may have to pay the price of efficiency. The reason is that far more moment conditions are incorporated than the existing estimators which only employ a finite fixed number of conditioning variables. More moment conditions, especially those which may not help much for the identification of the parameters or even redundant, will raise the asymptotic variance of our estimator definitely. A simulation study is actually conducted to confirm this conjection by a model defined via conditional moment restrictions with only one conditioning variable and hence for which both Domínguez and Lobato's(2004) and our estimators are consistent. However, our focus here is on the situation that the model is defined generically with an infinite dimensional conditional information set and no prior information exists for which variables should be included for the identification. That is, we concentrate on the identification problem and consistency of the estimator. Therefore, our estimator is a useful complement to the existing estimators like Hansen and Singleton's(1982) GMM instead of a substitute.

Our estimator is closely connected to the literature of testing martingale difference(MD) property and specification testing for time series models which are usually based on such frequency domain tools as power spectrum and generalized spectrum(Hong, 1999; Hong and Lee, 2005; Durlauf, 1991; Escanciano and Velasco, 2006; Deo, 2000; Escanciano, 2006). In fact, the conditional moment restrictions with infinite dimensional conditioning set which define the models we are estimating are the same as those for the MD property and specification null hypothesis for time series models. To test such conditions, Durlauf(1991) proposes checking the fact that the standardized spectral distribution function is a straight line which holds under these conditional moment restrictions. Later Deo(2000) extends Durlauf's(1991) test to allow for some types of conditional heteroskedasticity. These tests are obtained by checking whether the standard spectral distribution function is close enough to the straight line via certain criterions such as the Cramér-Von Mises distance. Correspondingly, such test statistics can be readily inverted into minimization criteria that can be used for estimation. Berkowitz(2001) is actually minimizing the test statistic in Durlauf(1991). However, similar to the discussions above, the tests based on standard power spectrum or spectral distribution function are only suitable for unconditional restriction-linear dependence and are not consistent against non-MD or mis-specification with zero autocorrelations which are purely non-linear dependence. Observing this, Hong(1999) proposes the generalized spectral density via a nonlinear dependence measure to check dependence structure of a time series. It is then extended by Hong and Lee(2005) to specification testing with conditional heteroskedasticity. The test statistic is based on a  $L^2$ -metric between smoothed generalized spectral density through kernel methods and the correspondents under the null hypothesis. Escanciano and Velasco(2006)

and Escanciano(2006), based on Hong(1999), suggest tests via the generalized spectral distribution function which replaces the standard spectral density in Durlauf(1991) by the generalized spectral density. Our estimator actually minimizes the Cramér-Von Mises test statistic in Escanciano and Velasco(2006) and Escanciano(2006) and hence captures the nonlinear dependence structure.

The proposed estimator is particularly suitable for estimating CCAPM which is generically defined by a conditional moment restriction with infinite dimensional conditional information set. In the literature for this model, an inconsistency has been well documented between calibration exercises and estimation results using traditional methods which only consider a fixed and finite number of conditioning variables (e.g., Ferson and Constantinides, 1991; Hansen and Singleton, 1996). Basically, a huge level of risk aversion, based on the calibration exercises of Mehra and Prescott (1985), is needed to match the equity premium observed for stock data while the estimated coefficient of risk aversion is pretty small relatively. This implies a serious equity risk-premium puzzle and casts strong doubt on the empirical plausibility of CCAPM. Motivated by the possibility that the existing estimators may not be consistent at all by only incorporating a fixed and finite number of conditioning variables in the infinite dimensional conditioning set, we estimate the CCAPM under the assumption of constant relative risk aversion by our proposed estimator and make comparisons to Hansen and Singleton's(1982) GMM. Contrary to the intuition and estimation results by GMM which assert a relatively small risk aversion to economic agents, the estimation evidence of our proposed estimator indicates a much larger coefficient of risk aversion. Although it still cannot account for the equity risk-premium completely, our estimator does represent a direction which can decrease the distance between the theoretical models and the empir-

ical evidences. Our thought is that combined with new theoretical efforts, our econometric methods could shed more lights on this puzzle.

## CHAPTER 2

### DETECTION OF ABRUPT STRUCTURAL CHANGES: A SPECTRAL APPROACH

#### 2.1 Literature review

My method contributes to two strands of related literature. First, it generalizes the detection of abrupt changes with unknown break dates for parameters in linear regression models by allowing occasional breaks to occur in a smooth change circumstance that is approximated by locally stationary processes. Second, my model complements the nonstationary literature in the frequency domain, which pays little attention to distinguishing the two types of structure changes.

There is a large literature related to abrupt changes with unknown break dates in coefficients in linear regression models.<sup>1</sup> Brown, Durbin, and Evans (1975) initially propose the cumulative sum (CUSUM) tests that are based on the maximum partial sum of the recursive residuals. Despite their wide appeal, Vogelsang (1999) shows that these tests have non-monotonic power, i.e., the power can decrease as the magnitude of the change increases. Alternatively, a class of tests exists that directly allows for breaks in the regression. Under the assumption of a single break, Quandt (1960) introduces a  $\sup F$  test to find the largest Chow statistic over all possible break dates. This test raises the problem that one parameter is only identified under the alternative, i.e., the break date; thus the limit distribution is unknown. Andrews (1993) and Andrews and Ploberger (1994) consider a supremum, the weighted exponential

---

<sup>1</sup>Perron (2006) provides an extensive review.



Lagrange multiplier (LM), the Wald, and the likelihood ratio (LR) tests for structural breaks with unknown break dates. The test statistics have nonstandard asymptotic distributions, and the critical values are obtained by simulations. This class of tests suffers from the non-monotonic power problem if the number of breaks present under the alternative is greater than the number of breaks explicitly accounted for in the construction of the tests. Therefore, Andrews, Lee, and Ploberger (1996) extend the weighted exponential statistics to multiple breaks. Bai and Perron (1998) propose a double maximum test given some upper bound and a sequential test. Qu and Perron (2007) extend the method to a system of equations. There is relatively less discussion about smooth changes in the linear regression framework. Farley, Hinich, and McGuire (1975) construct an  $F$  test against the parametric alternative with its slope being a linear function of time. Lin and Terasvirta (1994) introduce an LM-type test against the Smooth Transition Regression (STR) model whose intercept and slope are smoothly time-varying. Chen and Hong (2012) propose a nonparametric generalized Hausman test for no structural changes versus both smooth and abrupt changes without knowing the change points. However, this test is not able to distinguish between abrupt and smooth changes. Although my method is not regression-based, it generalizes the model setups of the regression models. In Section 3, I carefully illustrate the relation between the instability of parameters in regression models and nonstationarity. In conclusion, my method connects the literature from the two justifications by providing a way to accommodate both abrupt and smooth structural changes and to distinguish abrupt changes from a smooth changing or stationary structure.

The frequency domain methods also focus mainly on testing stationarity versus nonstationarity without differentiating between the two types of structural

changes. The seminal work on the frequency domain methods for nonstationarity is Priestley (1965) in which the conventional spectrum is generalized to a time-dependent spectrum. In his paper, the spectra are assumed to change smoothly over time for nonstationary processes. Dahlhaus (1996a, 1996b, 1997) develops the evolutionary spectrum to the local stationarity to derive asymptotic properties. Von Sachs and Neumann (2000) develop a test for stationarity based on empirical wavelet coefficients that are estimated by using localized versions of the periodogram. Pararodits (2009) proposes a test for stationarity against the alternative of a time-varying spectral structure. This test is based on a comparison between the sample spectral density and a global spectral density estimator. Dwivedi and Subba Rao (2011) test for stationarity based on the discrete Fourier transform. Picard (1985) estimates an abrupt structural change point by a generalized Kolmogorov-Smirnov statistic based on cumulative spectral density functions. My approach adopts local stationarity to approximate the smooth changing nonstationary process and relaxes its restriction of strictly continuous spectra by allowing occasional breaks. It also contributes to the frequency domain literature by providing a way to estimate and detect multiple abrupt changes.

## **2.2 Covariance structural changes versus parameter instability in regression models**

The spectrum-based approach is a nonparametric method that directly examines the changing patterns in a covariance structure via the time-varying spectra. However, in most parametric models, the nonstationarity of stochastic pro-

cesses due to the covariance structure can be represented by the changing patterns of parameters. Here, I consider the following linear regression model

$$Y_t = \mathbf{X}_t' \alpha_t + \varepsilon_t$$

where  $Y_t$  is a dependent variable,  $X_t$  is a  $d \times 1$  vector of explanatory variables,  $\alpha_t$  is a  $d \times 1$  parameter vector with a part or all of the elements time varying, and  $\varepsilon_t$  is an unobservable disturbance with  $E(\varepsilon_t | \mathbf{X}_t) = 0$ . The regressor vector  $\mathbf{X}_t$  can contain exogenous explanatory variables and lagged dependent variables. Suppose  $\mathbf{X}_t$  is a vector of exogenous explanatory variables, for simplicity, and let  $d = 2$ ,  $\mathbf{X}_t = (X_{1t}, X_{2t})'$ , and  $\alpha_t = (\alpha_{1t}, \alpha_{2t})'$ . Then

$$\begin{pmatrix} f^{X_1, Y}(t, \omega) \\ f^{X_2, Y}(t, \omega) \end{pmatrix} = \begin{pmatrix} f^{X_1}(t, \omega) & f^{X_1, X_2}(t, \omega) \\ f^{X_1, X_2}(t, \omega) & f^{X_2}(t, \omega) \end{pmatrix} \begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \end{pmatrix}$$

where  $f(t, \omega)$  is a spectrum at  $(t, \omega)$  and  $f^{\cdot\cdot}(t, \omega)$  is a cross-spectrum. In the existing time domain literature, the explanatory variables and disturbance are assumed to be stationary. This assumption means that the spectral matrix to the right is time independent, hence the investigation of time dependence of the coefficients is equivalent to the investigation of the cross spectra between the explanatory variables and the dependent variable. Therefore, the nonstationarity of the dependent variables is represented as the instability in the coefficients in the regression model. In contrast, by the spectral method I propose, the stationary assumption of explanatory variables and disturbance is relaxed so that the spectral matrix is time varying. As a result of this relaxation, the time dependence of the spectral matrix also attributes to the time varying of the parameters and vice versa. This also means that the instability of parameters in the models may be due to nonstationarity in both the dependent variable and explanatory variables.

When  $\mathbf{X}_t$  is a vector of lagged dependent variables,  $Y_t$  becomes an autoregressive process, and the coefficients involved in the covariance structure are moving through time. Setting  $d = 1$  and  $X_t = Y_{t-1}$ , for example, the regression is reduced to an AR(1) model. In this model,

$$\begin{aligned}\sigma_t(0) &= \alpha_t^2 \sigma_{t-1}(0) \\ \sigma_t(s+1) &= \alpha_{t+s} \sigma_t(s),\end{aligned}$$

where  $\sigma_t(s)$  is defined as an autocovariance with lag  $s$  at time  $t$  and is the variance at time  $t$  when  $s = 0$ . The equations show us that the time varying of the covariance structure is represented by changes of coefficients in time. In this AR(1) model, the underlying assumption is that the coefficients of lag orders greater than 1 are fixed as 0, and merely the first lag order is allowed to have a time variant coefficient. In contrast, frequency domain methods can examine infinite orders of the covariance structure without restrictions on the coefficients.

In addition to the regression models, the instability of the parameters in a simultaneous system of equations is also due to the time-varying covariance structure. For instance, the first order vector autoregression (VAR(1)) with a set of two dependent variables is defined as follows:

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \alpha_{11,t} & \alpha_{12,t} \\ \alpha_{21,t} & \alpha_{22,t} \end{pmatrix} \begin{pmatrix} X_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}.$$

Similar to AR(1), these are the equations between covariances:

$$\begin{aligned}\Sigma_t(0) &= \begin{pmatrix} \alpha_{11,t} & \alpha_{12,t} \\ \alpha_{21,t} & \alpha_{22,t} \end{pmatrix} \Sigma_{t-1}(0) \begin{pmatrix} \alpha_{11,t} & \alpha_{21,t} \\ \alpha_{12,t} & \alpha_{22,t} \end{pmatrix} \\ \Sigma_t(s+1) &= \begin{pmatrix} \alpha_{11,t+s} & \alpha_{12,t+s} \\ \alpha_{21,t+s} & \alpha_{22,t+s} \end{pmatrix} \Sigma_t(s),\end{aligned}$$

where  $\Sigma_t(s)$  is a covariance matrix at time  $t$ , defined as

$$\begin{pmatrix} \sigma_t^X(s) & \sigma_t^{X,Y}(s) \\ \sigma_t^{Y,X}(s) & \sigma_t^Y(s) \end{pmatrix}$$

and is a variance matrix at time  $t$  when  $s = 0$ . Therefore, the time-varying covariance structure leads to time-varying coefficients. Similar to AR(1), in VAR(1) only the set of first lag order coefficients is allowed to be time variant. In contrast, the spectral method is free of the time invariant assumption on the lag order parameters.

### 2.3 Abrupt structural changes and local stationarity

I consider a bivariate stochastic process  $X_{lt}$  with  $E(X_{lt}) = 0$  for  $l = 1, 2$ . Otherwise the mean is assumed not to be affected by changes in a covariance structure. Suppose there are  $m_l$  ( $m_l \geq 0$ ,  $l = 1, 2$ ) unknown abrupt structural changes. Without loss of generality, I denote the interval as  $\mathbf{T}_{k_l} = [T_{k_l-1}, T_{k_l})$  with  $k_l = 1, \dots, m_l + 1$ ,  $T_0 = 0$ ,  $T_{m_l+1} = T$ , and  $\cup\{\mathbf{T}_{k_l}\}_{k_l=1}^{m_l+1} = [0, T]$ . Furthermore, I denote the distance of the interval as  $|\mathbf{T}_{k_l}|$ . In addition, the corresponding abrupt breaks are rescaled as  $\lambda_{k_l} = \frac{T_{k_l}}{T}$  with  $k_l = 1, \dots, m_l + 1$ . The distance of  $\lambda_{k_l} = [\lambda_{k_l-1}, \lambda_{k_l})$  is denoted as  $|\lambda_{k_l}|$ . Then each sequence of stochastic processes  $X_{lt, T_k}$  ( $t \in \mathbf{T}_{k_l}$ ) has a smoothly changing structure or stationary structure. In the frequency domain, these abrupt changes are the step discontinuous points in time in the spectral density function, which is continuous otherwise.

In the rest of this section, I show that local stationarity can be used to approximate both the smooth changing covariance structure and the stationarity

of  $X_{lt, T_k}$ . Generally,  $X_{lt}$  can be rewritten in a time-varying spectral representation

$$X_{lt, T_k} = \int_{-\pi}^{\pi} e^{i\omega t} A_{lk} \left( \frac{t}{T}, \omega \right) dZ_l(\omega),$$

where  $Z_l(\omega)$  is a stochastic process on  $[-\pi, \pi]$  with  $Z_l^*(\omega) = Z_l(-\omega)$ . Hence  $A_{lk} : [\lambda_{k_l-1}, \lambda_{k_l}] \times \mathbb{R} \rightarrow \mathbb{C}$  is a  $2\pi$ -periodic function with  $A_{lk}(u, -\omega) = A_{lk}^*(u, \omega)$  and it is continuous. By definition of locally stationary processes (Dahlhaus 1996), there exists an  $A_{lt, T_k}^0(\omega)$  such that  $X_{lt, T_k}$  has the following representation:

$$X_{lt, T_k} = \int_{-\pi}^{\pi} e^{i\omega t} A_{lt, T_k}^0(\omega) dZ_l(\omega).$$

And for a constant  $K$ , there is

$$\sup_{t \in T_k, \omega} \left| A_{lt, T_k}^0(\omega) - A_{lk} \left( \frac{t}{T}, \omega \right) \right| \leq \frac{K}{T}.$$

Then  $X_{lt, T_k}$  is called locally stationary in  $T_{k_l}$  with the transfer function  $A_{lk}^0$ . The smoothness of  $A_{lk}$  in  $u$  guarantees that the process has a stationary behavior locally. Additional smoothness properties of  $A_{lk}$  are assumed in both components in the following theorem. I propose a spectrum for fixed  $T_{k_l}$  of the locally stationary process:

$$f_{T_k}(u, \omega) := \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \text{Cov}(X_{luT, T_k}, X_{l'uT-s, T_k}) \exp(-i\omega s).$$

I prove that  $f_{lT_k}(u, \omega)$  converges in squared mean to  $f_l(u, \omega) = f_{lk}(u, \omega) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \sigma_{ls}(u) \exp(-i\omega s)$  for each corresponding  $T_k$  via the following theorem.

**Theorem 1** *Let  $f_{T_k}^X$  be a spectrum for fixed  $T_k$ :*

$$f_{T_k}^X(u, \omega) := \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \text{Cov}(X_{luT, T_k}, X_{l'uT-s, T_k}) \exp(-i\omega s),$$

and  $f_k^X = A_{lk}^*(u, \lambda) A_{lk}(u, \lambda)$ . If  $X_{lt, T_k}$  is locally stationary and  $A_{lk}(u, \omega)$  is uniformly Lipschitz continuous in both components with  $\alpha > \frac{1}{2}$ , then for  $u \in (\lambda_{k-1}, \lambda_k)$ , there is

$$\int_{-\pi}^{\pi} |f_{T_k}^X(u, \omega) - f_k^X(u, \omega)|^2 d\omega = o(1). \quad (2.1)$$

Therefore,  $f_{T_k}(u, \omega)$  converges to  $f_k(u, \omega)$  in mean square. The proof implies that only the  $\varepsilon_{lt, T_k}$  with  $\frac{t}{T} \in [u - \frac{n}{T}, u + \frac{n}{T}]$ , the neighborhood of  $u$ , contributes to the spectrum  $f_k(u, \omega)$ . As  $\frac{n}{T_k} \rightarrow 0$ , to some extent the observations on the interval can be considered as stationary. In other words, for each  $u$  belonging to  $[\lambda_{k-1}, \lambda_k]$ , there exists an interval  $B_u \subset [\lambda_{k-1}, \lambda_k)$ , such that  $\varepsilon_{lt, T_k}$  with  $\frac{t}{T} \in B_u$  has a homogenous spectrum as  $X_{uT, T_k}$  and can be considered as stationary, and thus used in the estimation.

## 2.4 Estimation for limit spectra

When  $u$  is a break point, *e.g.*,  $u = \lambda_{k-1}$ , the spectrum is step discontinuous, with left limit  $f_{T_{k-1}}^-(\lambda_{k-1}, \omega)$  and right limit  $f_{T_k}^+(\lambda_{k-1}, \omega)$  unequal for some  $\omega \in [-\pi, \pi]$ . Without loss of generality, I let the spectrum be right continuous. Therefore,  $f_{T_{k-1}}^-(\lambda_{k-1}, \omega) = \lim_{u \nearrow \lambda_{k-1}} f_{T_{k-1}}(u, \omega)$  and  $f_{T_k}^+(\lambda_{k-1}, \omega) = \lim_{u \searrow \lambda_{k-1}} f_{T_k}(u, \omega) = f_{T_k}(\lambda_{k-1}, \omega)$ . To estimate the difference between the left and right limits of the spectral density at point  $u$ , thus detecting the changing points, only data on the left of point  $u$  are used to estimate the left limit and data on the right are used to estimate the right limit. Specifically, to estimate the two limits, I let  $n$  be a positive even integer such that  $0 < n < T$  and  $m = n/2 + 1$ , and define  $\mathbf{X}_l^- = (X_{luT-m}^-, X_{luT-m+2}^-, \dots, X_{luT-1}^-) = (X_{luT-m}, X_{luT-m+2}, \dots, X_{luT-1})$  and  $\mathbf{X}_l^+ = (X_{luT-m}^+, X_{luT-m+2}^+, \dots, X_{luT-1}^+) = (X_{luT}, X_{luT+1}, \dots, X_{luT+m-1})$ . Intuitively, the data to the right of  $u$  are symmetrized to the left. Then the local periodogram  $I_m^-(u - \frac{1}{T}, \omega)$  is calculated as the estimator of  $f_{T_{k-1}}(u - \frac{1}{T}, \omega)$ , and  $I_{m_u}^+(u, \omega)$  is calcu-

lated as the estimator of  $f_{T_k}(u, \omega)$ : For  $l, l' = 1, 2$ ,

$$\begin{aligned} I_m^-\left(u - \frac{1}{T}, \omega\right) &= \frac{1}{2\pi H_n} \left( \sum_{t=0}^{m-1} h_{t,m} X_{t+uT-m}^- e^{-i\omega t} \right) \left( \sum_{t=0}^{m-1} h_{t,m} X_{t'+uT-m}^- e^{i\omega t} \right) \\ I_m^+(u, \omega) &= \frac{1}{2\pi H_n} \left( \sum_{t=0}^{m-1} h_{t,m} X_{t+uT-m}^+ e^{-i\omega t} \right) \left( \sum_{t=0}^{m-1} h_{t,m} X_{t'+uT-m}^+ e^{i\omega t} \right) \end{aligned}$$

where  $h_{t,n}$  is a taper function, e.g.,  $h_{t,n} = h\left(\frac{t}{n}\right) = \frac{1}{2} \left[ 1 - \cos\left(2\pi \frac{t}{n}\right) \right]$  and  $H_n = \sum_{t=0}^{m-1} h_{t,n}^2$ . The data are smoothed by the taper function: the observations near the point  $u$  with weights approaching 1 and those farther away with less weights. In addition, the use of the taper function reduces not only the bias of a periodogram due to the well-known leakage effect but also the bias due to the nonstationarity of the processes. Since this periodogram is not a consistent estimator, it needs to be smoothed in its frequency domain, leading to the kernel estimator

$$\begin{aligned} \widehat{I}_m^-\left(u - \frac{1}{T}, \omega\right) &= \frac{1}{b} \int_{-\pi}^{\pi} K\left(\frac{\omega - \mu}{b}\right) I_m^-\left(u - \frac{1}{T}, \mu\right) d\mu \\ \widehat{I}_m^+(u, \omega) &= \frac{1}{b} \int_{-\pi}^{\pi} K\left(\frac{\omega - \mu}{b}\right) I_m^+(u, \mu) d\mu \end{aligned}$$

where  $K$  is a kernel with compact support  $[-1, 1]$  satisfying  $K(x) = K(-x)$  and  $\int K(x) dx = 1$ , and  $b$  is the bandwidth in the frequency domain.

## 2.5 Test statistic and its asymptotic distribution

In this section, I construct a Kolmogorov-Smirnov type statistic to test the null hypothesis of no abrupt change at the point  $u$ ,

$$H_0 : f^-(u, \omega) = f^+(u, \omega) \quad \forall \omega \in [-\pi, \pi]$$

versus the alternative hypothesis that  $u$  is a changing point, i.e.,

$$H_A : f^-(u, \omega) \neq f^+(u, \omega) \text{ for some } \omega \in [-\pi, \pi] \text{ with a positive measure.}$$



For a fixed  $u$ ,  $f^\pm(u, \cdot)$  has the following properties:

- i)  $\int_{-\pi}^{\pi} f^\pm(u, \omega) d\omega = \sigma_0(u)$ , where  $\sigma_0(u)$  is a variance or contemporary cross-covariance;
- ii)  $f^\pm(u, \omega) \geq 0 \forall \omega \in [-\pi, \pi]$  when  $f^\pm(u, \omega)$  is a spectral density;
- iii)  $f^\pm(u, \omega) = f^\pm(u, -\omega) \forall \omega \in [-\pi, \pi]$ .

I further define  $F^\pm(u, \nu) = \int_0^\nu f^\pm(u, \omega) d\omega$ . Then  $F^\pm(u, \nu)$  is monotonically increasing and right continuous on  $\nu$ . Therefore, testing  $H_0$  versus  $H_A$  is equivalent to testing

$$H'_0 : F^-(u, \nu) = F^+(u, \nu) \quad \forall \nu \in [0, \pi]$$

versus

$$H'_A : F^-(u, \nu) \neq F^+(u, \nu) \text{ for some } \nu \in [0, \pi].$$

The corresponding empirical spectral cumulative distribution functions for observations to the left and right limits can be defined as follows, respectively:

$$\begin{aligned} \widehat{F}_m^-\left(u - \frac{1}{T}, \nu\right) &= \int_0^\nu \widehat{I}_m^-\left(u - \frac{1}{T}, \omega\right) d\omega \\ \widehat{F}_m^+(u, \nu) &= \int_0^\nu \widehat{I}_m^+(u, \omega) d\omega. \end{aligned}$$

By Dahlhaus (1985), for a fixed  $u$ , under the null hypothesis, in  $D[0, \pi]$ ,

$$\sqrt{m} \left( \left| \widehat{F}_m^+(u, \nu) \right| - \left| F^+(u, \nu) \right| \right) \Longrightarrow Z(\nu),$$

where  $Z(\nu)$  is a Gaussian process with

$$E(Z(\nu)) = 0$$

and

$$\text{cov}(Z(\nu), Z(\nu')) = 2\pi \left( \left| J(\nu \wedge \nu') + F_4(\nu, \nu') \right| \right),$$

where

$$J(v) := \int_0^v f^{+2}(u, \omega) d\omega$$

and

$$F_4(v, v') = \int_0^v \int_0^{v'} f_4^+(\alpha, -\alpha, -\beta) d\alpha d\beta.$$

$f_k^+$  is the  $k$ th order cumulative spectrum. All moments of  $\{X_t\}$  up to the 8th order are assumed to exist and the 4th order cumulant is defined as

$$\begin{aligned} c_4(s_1, s_2, s_3) = & E(X_t X_{t+s_1} X_{t+s_2} X_{t+s_3}) - E(X_t X_{t+s_1}) E(X_{t+s_2} X_{t+s_3}) \\ & - E(X_t X_{t+s_2}) E(X_{t+s_1} X_{t+s_3}) - E(X_t X_{t+s_3}) E(X_{t+s_1} X_{t+s_2}). \end{aligned}$$

**Assumption 2.1** For  $j = 1, 2, 3$ , I have  $\sum_{s_1=-\infty}^{\infty} \sum_{s_2=-\infty}^{\infty} \sum_{s_3=-\infty}^{\infty} |s_j c_4(s_1, s_2, s_3)| < \infty$ .

$$f_4(\omega_1, \omega_2, \omega_3) = \frac{1}{(2\pi)^3} \sum_{s_1=-\infty}^{\infty} \sum_{s_2=-\infty}^{\infty} \sum_{s_3=-\infty}^{\infty} c_4(s_1, s_2, s_3) e^{-i(\sum_{j=1}^3 \omega_j s_j)}.$$

Hence  $\int_0^{v'} f_4^+(\alpha, -\alpha, -\beta)$  is defined with respect to  $c_4^+(s_1, s_2, s_3)$ . On the other hand, by the same argument,

$$\sqrt{m} \left( \left| \widehat{F}_m^-(u - \frac{1}{T}, v) \right| - |F^-(u, v)| \right) \Rightarrow Z(v).$$

Therefore, similar to the two sample Kolmogorov-Smirnov test, under the null hypothesis,

$$\sqrt{m/2} \left( \left| \widehat{F}_m^+(u, v) \right| - \left| \widehat{F}_m^-(u - \frac{1}{T}, v) \right| \right) \Rightarrow Z(v).$$

I denote

$$Q_m(u) = \sqrt{\frac{m}{2}} \sup_{v \in [0, \pi]} \left| \left| \widehat{F}_m^+(u, v) \right| - \left| \widehat{F}_m^-(u - \frac{1}{T}, v) \right| \right|$$

and

$$Q(u) = \sup_{v \in [0, \pi]} |Z(v)|.$$

By the continuous mapping theorem,

$$Q_m(u) \Rightarrow Q(u).$$

The test of the null hypothesis is constructed by using the critical values of the distribution of  $Q(u)$ . The null hypothesis is rejected at level  $\alpha$  if  $Q_m(u) > K_\alpha$ , where  $K_\alpha$  is found from  $P(Q(u) \leq K_\alpha) = 1 - \alpha$ . In practice, the level of the critical values can be approximated, when  $m$  is large, by

$$P(\widehat{Q}(u) > K_\alpha)$$

where  $K_\alpha$  is the one-tailed critical value of the asymptotic distribution of  $\widehat{Q}(u)$  at level  $\alpha$ . In this case,  $Q(u)$  has been replaced by its consistent estimator  $\widehat{Q}(u)$  defined as  $\sup_{v \in [0, \pi]} |\widehat{Z}(v)|$ .  $\widehat{Z}(v)$  is a normal distribution with the mean of 0, and with the variance  $2\pi \left( |\widehat{J}(v) + \widehat{F}_4(v, v)| \right)$ :

$$\begin{aligned} \widehat{J}(v) &= \int_0^v \left( \widehat{I}_m^+(u, \omega) \right)^2 d\omega \\ \widehat{F}_4(v, v) &= \int_0^v \int_0^v \widehat{f}_4^+(\alpha, -\alpha, -\beta) d\alpha d\beta, \end{aligned}$$

where

$$\widehat{f}_4^+(\alpha, -\alpha, -\beta) = \frac{1}{(2\pi)^3} \sum_{|s_1|=0}^{m-1} \sum_{|s_2|=0}^{m-1} \sum_{|s_3|=0}^{m-1} \widehat{c}_4^+(s_1, s_2, s_3) e^{-i((s_1-s_2)\alpha + s_3\beta)}$$

with

$$\begin{aligned} \widehat{c}_4^+(s_1, s_2, s_3) &= \frac{1}{\sum_{t=0}^{m-1} h_{t,n}^4} \sum_{t=0}^{m-1} \left( h_{t,0} h_{t,s_1} h_{t,s_2} h_{t,s_3} X_t^+ X_{t+s_1}^+ X_{t+s_2}^+ X_{t+s_3}^+ \right) \\ &\quad - \frac{1}{H_n^2} \left[ \sum_{t=0}^{m-1} h_{t,0} h_{t,s_1} X_t^+ X_{t+s_1}^+ \sum_{t=0}^{m-1} h_{t,s_2} h_{t,s_3} X_{t+s_2}^+ X_{t+s_3}^+ \right. \\ &\quad - \sum_{t=0}^{m-1} h_{t,0} h_{t,s_2} X_t^+ X_{t+s_2}^+ \sum_{t=0}^{m-1} h_{t,s_1} h_{t,s_3} X_{t+s_1}^+ X_{t+s_3}^+ \\ &\quad \left. - \sum_{t=0}^{m-1} h_{t,0} h_{t,s_3} X_t^+ X_{t+s_3}^+ \sum_{t=0}^{m-1} h_{t,s_1} h_{t,s_2} X_{t+s_1}^+ X_{t+s_2}^+ \right]. \end{aligned}$$

Therefore,

$$\begin{aligned}
\widehat{F}_4(v, v) &= \frac{1}{(2\pi)^3} \sum_{|s_1|=0}^{m-1} \sum_{|s_2|=0}^{m-1} \sum_{|s_3|=0}^{m-1} \int_0^v \int_0^v \widehat{c}_4^+(s_1, s_2, s_3) e^{-i((s_1-s_2)\alpha+s_3\beta)} d\alpha d\beta \\
&= \frac{1}{(2\pi)^3} \left[ \left( \frac{1}{\sum_{t=0}^{m-1} h_{t,n}^4} - \frac{m}{\left(\sum_{t=0}^{m-1} h_{t,n}^2\right)^2} \right) \left( \int_0^v \left| \sum_{t=0}^{m-1} h_{t,m} X_{t+uT-m}^+ e^{-i\alpha t} \right|^2 d\alpha \right)^2 \right. \\
&\quad \left. - \frac{\sum_{t=0}^{m-1}}{\left(\sum_{t=0}^{m-1} h_{t,n}^2\right)^2} \int_{-v}^v e^{-i\alpha t} \left( \sum_{t=0}^{m-1} h_{t,m} X_{t+uT-m}^+ e^{i\alpha t} \right) \left| \sum_{t=0}^{m-1} h_{t,m} X_{t+uT-m}^+ e^{-i\alpha t} \right|^2 d\alpha \right. \\
&\quad \left. \int_0^v e^{-i\beta t} \left( \sum_{t=0}^{m-1} h_{t,m} X_{t+uT-m}^+ e^{i\beta t} \right) d\beta \right].
\end{aligned}$$

I investigate the second term first. In the first half of the integrand,

$$e^{-i\alpha t} \left( \sum_{t_1=0}^{m-1} h_{t,m} X_{t_1+uT-m}^+ e^{i\alpha t_1} \right) = \sum_{t_1=0}^{m-1} h_{t_1,m} X_{t_1+uT-m}^+ (\cos(\alpha s_1) + i \sin(\alpha s_1)).$$

The second half of the integrand can be rewritten as  $\sum_{|s|=0}^{m-1} \varphi(s) (\cos(\alpha s_1) + i \sin(\alpha s_1))$ ,

where  $\varphi(s) := \sum_t h_{t,m} X_t^+ X_{t-s}^+$ . Hence, the integrand is as follows:

$$\begin{aligned}
&\sum_{|s|=0}^{m-1} \sum_{t_1=0}^{m-1} \varphi(s) h_{t_1,m} X_{t_1+uT-m}^+ (\cos(\alpha s_1) + i \sin(\alpha s_1)) (\cos(\alpha s_1) + i \sin(\alpha s_1)) \\
&= \sum_{|s|=0}^{m-1} \sum_{t_1=0}^{m-1} \varphi(s) h_{t_1,m} X_{t_1+uT-m}^+ (\cos(\alpha(s_1 + s)) - i \sin(\alpha(s_1 + s))).
\end{aligned}$$

Therefore, by taking integration with respect to  $\alpha$ , the first term becomes

$$\Delta(t, v) = 2 \sum_{t_1=0}^{m-1} \sum_{|s|=0}^{m-1} \varphi(s) h_{t_1,m} X_{t_1+uT-m}^+ \frac{\sin((t_1 - t + s)v)}{t_1 - t + s}.$$

For the integration with respect to  $\beta$ ,

$$\begin{aligned}
&\sum_{|s_2|=0}^{m-1} \sum_{t=0}^{m-1} \Delta(t, v) \int_0^v h_{t_2,m} X_{t_2+uT-m}^+ (\cos(\beta s_2) + i \sin(\beta s_2)) d\beta \\
&= \sum_{s_2=1-m}^{m-1} \sum_{t=0}^{m-1} \Delta(t, v) h_{t_2,m} X_{t_2+uT-m}^+ \frac{\sin(vs_2)}{s_2},
\end{aligned}$$

where the imaginary part cancels by the symmetry of  $|s_2|$ .

## 2.6 Estimation for abrupt changes

### 2.6.1 Estimation for A Single Change

In this section, I construct an estimator for the single abrupt change based on the statistic  $Q_m(u)$  that essentially is a comparison of the estimators of the left and right spectra. Intuitively, among time spots which have been rejected for the null hypothesis that the point is not an abrupt changing point, the location of the maximum of the differences could be a reasonable estimator for the location of the abrupt change. To estimate the inner abrupt change, first I impose the restriction that the changing point is asymptotically bounded from the boundaries of the sample. That is, I define an arbitrary small positive number  $\epsilon > 0$  such that for the true changing point  $\lambda$ , there is  $\lambda \in (\epsilon, 1 - \epsilon)$ . Then I define the estimator

$$\widehat{\lambda} = \arg \max_{m/T \leq u \leq 1-m/T} Q_m(u) \sqrt{2/m}.$$

To investigate the asymptotic distribution of  $\widehat{\lambda}$ , I let

$$\begin{aligned} D_m(z) &= 2/m Q_m\left(\lambda + \frac{mz}{T}\right)^2 \\ &= \left\| \widehat{F}_m^+\left(\lambda + m\frac{z}{T}\right) - \widehat{F}_m^-\left(\lambda + m\frac{z}{T} - \frac{1}{T}\right) \right\|^2 \\ &= \left\| \widehat{F}_m^+\left(\lambda + m\frac{z}{T}, \nu^*\right) - \widehat{F}_m^-\left(\lambda + m\frac{z}{T} - \frac{1}{T}, \nu^*\right) \right\|^2 \\ &= \left[ \sup_{\nu} \left\| \widehat{F}_m^+\left(\lambda + m\frac{z}{T}, \nu\right) - \widehat{F}_m^-\left(\lambda + m\frac{z}{T} - \frac{1}{T}, \nu\right) \right\| \right]^2 \end{aligned}$$

and define for some  $0 < M < \infty$ ,  $-M \leq z \leq M$ , the sequence of stochastic processes as

$$\zeta_n(z) = \frac{T}{m} \left( D_m\left(\frac{m}{T}z\right) - D_m(0) \right).$$

$\zeta_n(z)$  is continuous on  $[-M, M]$ . Furthermore, I define

$$\delta_{t_1, t_2, s, \tau}(u) = E \left( X_{t_1}^+ X_{t_1-s}^+ - X_{t_1}^- X_{t_1-s}^- \right) \left( X_{t_2}^+ X_{t_2-\tau}^+ - X_{t_2}^- X_{t_2-\tau}^- \right)$$

and assume that it is a continuous function of  $u$  and is infinitely differentiable in a neighborhood of  $\lambda$ . I have

**Theorem 2**  $\sqrt{\frac{m}{T}} (\zeta_n(z) - K_1 z) \xrightarrow{d} N(0, K_2 z)$ , where

$$K_1 = \frac{1}{4\pi^2 (H_n)^2} \sum_{|s|=0}^{\infty} \sum_{|\tau|=0}^{\infty} \phi_{Re}(s) \phi_{Re}(\tau) \sum_{t_1=|s|}^{\infty} \sum_{t_2=|\tau|}^{\infty} h_{t_1} h_{t_1-s} h_{t_2} h_{t_2-\tau} \delta'_{t_1, t_2, s}(\lambda),$$

and

$$K_2 = \sum_{|s|=0}^{\infty} \sum_{|\tau|=0}^{\infty} \sum_{|s'|=0}^{\infty} \sum_{|\tau'|=0}^{\infty} \left( \int_0^{\nu^*} \cos \omega s d\omega \int_0^{\nu^*} \cos \omega \tau d\omega + \int_0^{\nu^*} \sin \omega s d\omega \int_0^{\nu^*} \sin \omega \tau d\omega \right) \left( \int_0^{\nu^*} \cos \omega s' d\omega \int_0^{\nu^*} \cos \omega \tau' d\omega + \int_0^{\nu^*} \sin \omega s' d\omega \int_0^{\nu^*} \sin \omega \tau' d\omega \right) (g_1'(\lambda) - 2g_2'(\lambda)),$$

with

$$g_1(u) = \text{Cov}(\widehat{\psi}_s(u) \widehat{\psi}_\tau(u), \widehat{\psi}_{s'}(u) \widehat{\psi}_{\tau'}(u))$$

and

$$g_2(u) = \text{Cov}(\widehat{\psi}_s(u) \widehat{\psi}_\tau(u), \widehat{\psi}_{s'}(\lambda) \widehat{\psi}_{\tau'}(\lambda)).$$

Then the asymptotic distributions of the estimated changing points  $\widehat{\lambda}$  can be obtained via the functional limit theorem. I denote  $Y = \frac{\sqrt{\frac{m}{T}}(\zeta_n(z) - K_1 z)}{K_2 z}$ , and then  $Y \xrightarrow{d} N(0, 1)$ . The process  $\zeta(z) = K_1 z + Y \sqrt{|z|}$  has the unique maximum at

$$z^* = \frac{T K_2 Y^2}{m 4 K_1^2}.$$

Let  $z_n$  be the maximizer of the process  $\zeta_n(z)$ . By construction,

$$\widehat{\lambda} = \lambda + \frac{m}{T} z_n.$$

The continuous mapping theorem implies that  $z_n \xrightarrow{d} z^*$ . Therefore,

$$4K_1^2 \frac{(\widehat{\lambda} - \lambda)}{K_2} \xrightarrow{d} \chi_{(1)}^2,$$

where  $(\widehat{\lambda} - \lambda)/K_2 > 0$ .

**Theorem 3**  $\widehat{\lambda}$  is a consistent estimator for  $\lambda$ , i.e., for any small number  $\eta > 0$ ,

$$\lim_{T \rightarrow \infty} P\left(|\widehat{\lambda} - \lambda| > \eta\right) = 0.$$

The above estimation is based on the assumption that there exists a single abrupt change in the sample or based on the test results from the previous section that the null hypothesis of not being an abrupt changing point is rejected for at least one time instant  $u$ . Furthermore, this procedure can be extended to time series data in which there exists at most one abrupt change.

**Assumption 2.2**  $Q$  is a continuous function in  $u$ .

Under this assumption, as  $\widehat{\lambda} \xrightarrow{p} \lambda$ ,  $Q(\widehat{\lambda}) \xrightarrow{p} Q(\lambda)$  by the continuous mapping theorem. Then  $Q(\widehat{\lambda})$  is used to test the null hypothesis that  $\widehat{\lambda}$  is not an abrupt change. If the true abrupt change exists and is at  $\lambda$ , then  $Q(\lambda)$  is large enough such that  $\lambda$  is rejected by the hypothesis test.  $\widehat{\lambda}$  converges to  $\lambda$  as the sample size  $T$  increases. Given  $\lambda$  the changing point,  $Q(\widehat{\lambda})$  converges to  $Q(\lambda)$ , and  $\widehat{\lambda}$  is likely to be rejected and considered as an estimated abrupt changing point. Otherwise, if there is no abrupt change,  $\widehat{\lambda}$  converges to a  $\lambda$  maximizing  $Q(u)$  for  $(\epsilon, 1 - \epsilon)$  but at this  $\lambda$  the test fails to be rejected. As a result,  $\widehat{\lambda}$  is more likely to be accepted as not being an abrupt changing point. In conclusion, the above argument can be stated as the following theorem.

**Theorem 4** *Given the information that there is at most one abrupt change, if the null hypothesis that  $\widehat{\lambda}$  is not an abrupt change is rejected, then  $\widehat{\lambda}$  detects an abrupt change and is the estimated abrupt change; otherwise, no abrupt change is detected.*

In a similar way, the estimation of the single abrupt change can be extended to that of multiple abrupt changes by requiring some additional assumptions. This is discussed in the next section.

## 2.6.2 Estimation for multiple abrupt changes

In this section, the estimation of the single abrupt change is applied to data that contain multiple abrupt changes. In the estimation of a single abrupt change, a data sample is assumed to have at most a single abrupt change. To apply the estimation procedure to a data sample with multiple abrupt changes, I impose an additional restriction to make the changing points not only asymptotically bounded from the boundaries of the sample but also distinct from each other. That is, given  $M$  abrupt changes, there exists an arbitrary small number  $\epsilon > 0$ , such that  $\Lambda_{\epsilon, \epsilon'} = \{(\lambda_1, \dots, \lambda_M) : \min_{1 \leq k \leq M} |\lambda_{k+1} - \lambda_k| > \epsilon', \lambda_1 > \epsilon, \lambda_M < 1 - \epsilon\}$ . Based on these restrictions, I segment the data  $\{t = m + 1, m + 2, \dots, T - m - 2, T - m - 1\}$  evenly with the same number of observations  $\Delta$  such that every segment contains at most one abrupt change. I denote the number of segments by  $n_\Delta$ , and then  $T = n_\Delta \Delta + 2m$ .

**Assumption 2.3** For every  $\epsilon' > 0$ , there exist a  $\Delta$  such that  $\frac{\Delta}{T} \leq \epsilon'$ .

**Assumption 2.4**  $\Delta \rightarrow \infty$  and  $\frac{\Delta}{T} \rightarrow \frac{1}{n_\Delta}$  as  $T \rightarrow \infty$ .



The first assumption ensures that in each segment there is at most one abrupt change. The intuition behind the second assumption is that the number of observations in each segment increases with the increase of the distance  $|T_{k+1} - T_k|$  between each abrupt change, when the sample size  $T$  increases. Also, when  $\Delta$  is large enough, the test for each estimated abrupt changing point can be considered as independent. Furthermore, once the number of segments is determined, the ratio of the segments  $\frac{\Delta}{T}$  is fixed asymptotically. By this procedure of estimation, all abrupt changes are tested and estimated with the same power and the same size. Then the single abrupt change estimation is subsumed as  $n_\Delta = 1$ . Therefore, I define the  $i$ th segment as  $\Delta_i$  and

$$\widehat{\lambda}_{n_i} = \arg \max_{u \in \Delta_i} Q_m(u) \sqrt{2/m}.$$

Similar to the single abrupt change estimation,  $\widehat{\lambda}_{n_i}$  converges in probability to  $\lambda_{n_i}$ , which maximizes  $Q(u)$  in the same segment  $\Delta_i$ . When  $\lambda_{n_i}$  is an abrupt changing point, i.e.,  $\lambda_{n_i} = \lambda_k$  for  $k \in [1, M]$ ,  $\widehat{\lambda}_{n_i}$  is to be rejected for the null hypothesis of not being an abrupt change. Otherwise, when  $\lambda_{n_i}$  is not an abrupt change, the test is apt to fail to reject the null hypothesis at  $\widehat{\lambda}_{n_i}$ . Based on this justification mechanism, we can detect the abrupt changes whether the number of changes are known or unknown.

First, I consider the case that the number of abrupt changes is known. For example, I assume that there are two abrupt changes  $\lambda_1$  and  $\lambda_2$ , and  $\epsilon < \lambda_1 < \lambda_2 < 1 - \epsilon$ . If the number of segments is correctly specified,  $\lambda_1$  and  $\lambda_2$  maximize  $Q(u)$  in their own segments, respectively. Therefore, I denote these two segments as  $\Delta_i$  and  $\Delta_{i'}$ , with  $i, i' \leq n_\Delta$  and  $i \neq i'$ . Among the interior points in  $\Delta_i$ , there exists a  $\widehat{\lambda}_1$  as a consistent estimator of  $\lambda_1$ . In the same way, the estimator for  $\lambda_2$  in  $\Delta_{i'}$  can be located.

In the estimation of two known abrupt changes, the number of segments is closely related to the locations of the changes and the distance between them, i.e.,  $|\lambda_1 - \lambda_2|$ . If the distance is small, a small number of segments  $n_\Delta$  may include both changing points in the same segments. For example, if  $\lambda_1 = \frac{1}{6}$  and  $\lambda_2 = \frac{3}{4}$ , then they are relatively far away from each other, with the distance of  $\frac{7}{12}$ . In this case,  $n_\Delta = 2$  is able to separate the changing points into different segments, and  $\frac{\Delta}{T} = \frac{1}{2} - \frac{m}{T} < \frac{7}{12}$ . However, if  $\lambda_1 = \frac{1}{6}$  and  $\lambda_2 = \frac{5}{12}$ , then the distance is  $\frac{1}{4}$ . In this case,  $n_\Delta = 2$  will include both changing points into the first segment, and the one with the larger  $Q_m(u)$  will be considered as the estimator of one of the changing point and the other changing point is unable to be detected. On the other hand,  $n_\Delta = 3$  can allocate the two changes to the first and second segments with  $\frac{\Delta}{T} = \frac{1}{3} - \frac{2m}{3T}$ . In a third example, I assume  $\lambda_1 = \frac{5}{12}$  and  $\lambda_2 = \frac{2}{3}$ , so the distance is  $\frac{1}{4}$  as in the previous example. However, in this case  $n_\Delta = 3$  includes both changes in the second segment. As indicated in the assumption,  $n_\Delta = 5$  ensures the two changes with the distance of  $\frac{1}{4}$  to be allocated in distinct segments for all possible interior locations. These examples verify the necessity of the assumption,  $\frac{\Delta}{T} = \frac{1}{5} - \frac{2m}{5T} < \frac{1}{4}$ . As a result, with the information for the number of abrupt changes  $m$ , the determination of  $n_\Delta$  is data-driven, starting from  $m$  and terminating when  $m$  estimators are located in distinct segments. All these estimators are consistent to the true abrupt changes, just as in the estimation of a single abrupt change.

Second, I consider the case of an unknown number of abrupt changes, which is more pervasive in practice. Compared to the case of the known number of changes, the difficulty lies in that whether all changes are allocated in different segments. Therefore, I can presume the value of  $\epsilon'$ , for example, as 0.05 as adopted by Bai and Perron (1998). Then  $n_\Delta$  is chosen as the smallest number such that  $\frac{\Delta}{T} = \frac{1}{n_\Delta} - \frac{2m}{n_\Delta T} < \epsilon'$ . Once the number of segments is determined,

the abrupt changes are estimated in each segment. The estimates of the abrupt changes are those time instants which maximize the statistics in their segments and at which the null hypothesis of not being an abrupt change is rejected in the test proposed in the previous section.

## 2.7 Finite-sample performance

To examine the finite sample performance, I consider the following data generating processes based on the class of linear regression models

$$Y_t = \alpha_{0t} + \alpha_{1t}X_t + \epsilon_t$$

with  $\epsilon_t \sim i.i.d.N(0, 1)$ . Additionally, I assume  $X_t = 0.5X_{t-1} + \varepsilon_t$  with  $\varepsilon_t \sim i.i.d.N(0, 1)$ .

### DGP 0 [No structure change]

$$Y_t = 1 + 1.2X_t + \epsilon_t.$$

### DGP 1 [A structural break (level shift) due to nonstationarity of $X_t$ ]

$$Y_t = 1 + 1.2X_t + \epsilon_t$$

with

$$X_t = \begin{cases} 0.6X_{t-1} - 0.2X_{t-2} + \varepsilon_t, & t \leq 0.5T \\ -0.2X_{t-1} + \varepsilon_t, & t > 0.5T \end{cases}.$$

### DGP 2 [A structural break (level shift)]

$$Y_t = \begin{cases} 1 + 1.2X_t + \epsilon_t, & t \leq 0.5T \\ 1 + 0.3X_{t-1} + \epsilon_t, & t > 0.5T \end{cases}.$$

### DGP 3 [Smooth structural change]

$$Y_t = 1 - 0.9 \sqrt{t/T} X_t + \epsilon_t$$

### DGP 4 [An abrupt change followed by a smooth change]

$$Y_t = \begin{cases} 1 + 1.2X_t + \epsilon_t, & t \leq 0.5T \\ 1 - 0.8 \sqrt{t/T} X_t + \epsilon_t, & t > 0.5T \end{cases}$$

Table 2.1: Simulation Results of  $\widehat{\lambda}$  at 5%

		X	Y	Cross
DGP0	Mean	0.5040	0.5031	0.4966
	Std	0.1466	0.1480	0.1477
DGP1	Mean	0.4642	0.4694	0.4634
	Std	0.0785	0.1044	0.0815
DGP2	Mean	0.4946	0.4721	0.4585
	Std	0.1475	0.0531	0.0678
DGP3	Mean	0.4930	0.5139	0.5291
	Std	0.1549	0.1466	0.1510
DGP4	Mean	0.4975	0.4623	0.4465
	Std	0.1472	0.0669	0.0835

Note: The  $\widehat{\lambda}$ s are filtered by the 5% significance level in the test. When there is no abrupt change in a sequence or in the system,  $\widehat{\lambda}$  has a mean of around 0.5 and a large standard deviation. When there is an abrupt change, the sample mean is close to the true changing point and the sample standard deviation is much smaller, indicating that the estimated abrupt changes are near the true changes.

The symmetric local periodogram implemented as the spectral estimator in these simulations depends on two "smoothing" parameters. The first is the parameter  $b$ , the bandwidth in the frequency direction. Second, I define  $b_t =$

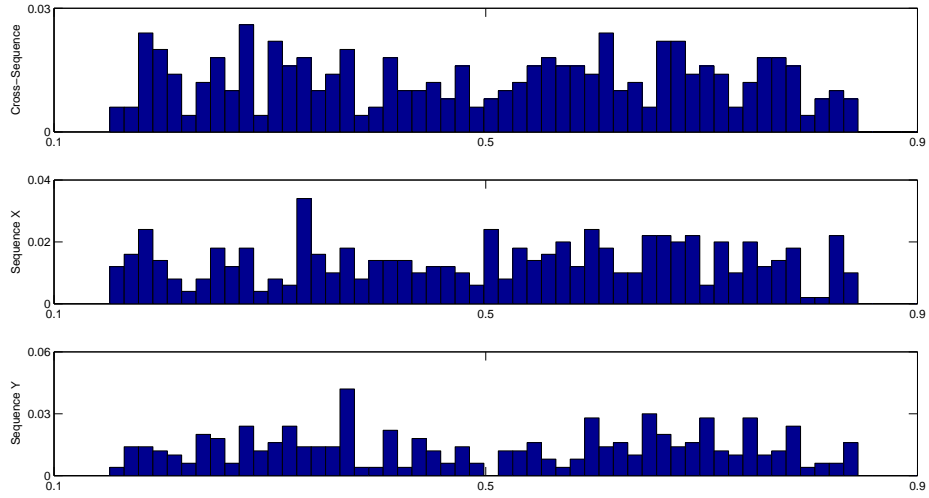


Figure 2.1: DGP 0: Frequency Distribution of  $\hat{\lambda}$

Without an abrupt change in the bivariate process,  $\hat{\lambda}$ s are distributed uniformly and have very small likelihoods.

$m/T$  as Dahlhaus (1996). Then  $b_t$  can be considered as the bandwidth in the time direction. As shown by Dahlhaus (1996b, Theorem 2.3), as  $T \rightarrow \infty$ ,  $b \sim T^{-1/6}$  and  $b_t \sim T^{-1/6}$ , i.e.,  $m \sim T^{5/6}$ . In the simulations, I let  $T = 800$  and iterate each data generating process for 500 times. In addition, by the data-driven method, the optimal  $b^* = 4/5T^{-1/6} = 0.26$  and the optimal  $m^* = 2/5T^{5/6} = 105$ .

To examine the size of the test under the null hypothesis that a potential point is not an abrupt change, I consider two data generating processes, DGP 0 and DGP 3, in both of which there are no abrupt changes. In DGP 0, the bivariate process is stationary; while in DGP 3, the covariance structure of  $Y_t$  is smoothly changing and so is the cross-covariance structure of the bivariate process.

To investigate the power of the test in detecting abrupt changes under different circumstances, I consider three other data generating processes: DGP1 with a structural break due to nonstationarity of  $X_t$ , i.e., the AR(1) process  $X_t$  has a

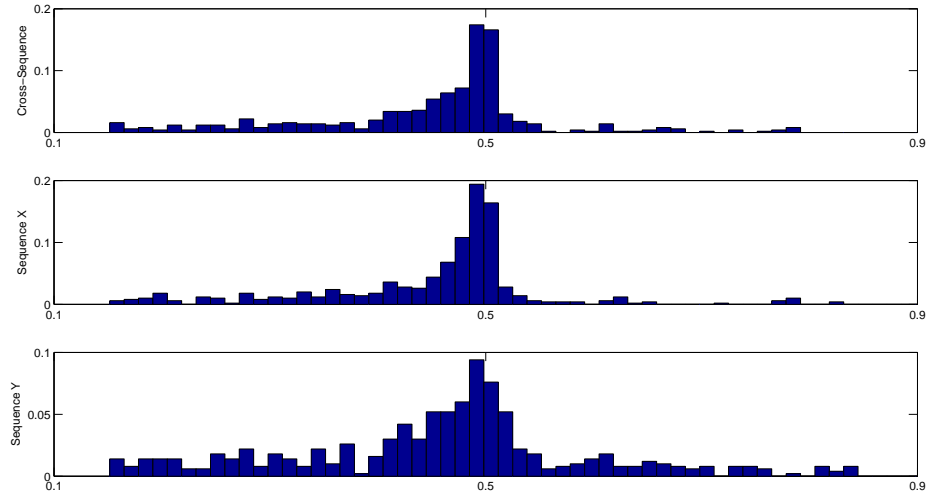


Figure 2.2: DGP 1: Frequency Distribution of  $\hat{\lambda}$

For the abrupt change at  $0.5T$  in the bivariate process,  $\hat{\lambda}$ s peak around the true changing points with large likelihoods.

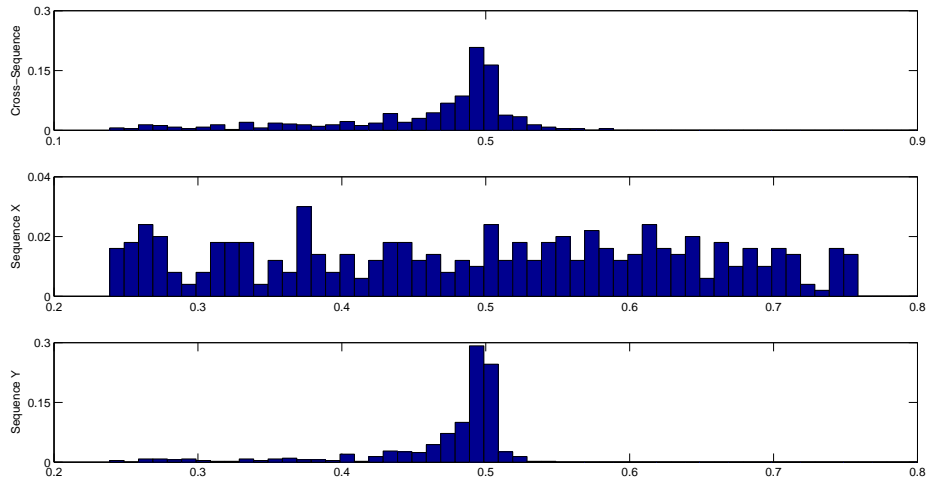


Figure 2.3: DGP 2: Frequency Distribution of  $\hat{\lambda}$

Without an abrupt change in the  $X_t$  process,  $\hat{\lambda}$ s are distributed uniformly and have very small likelihoods. For abrupt changes at  $0.5T$  in the covariance structure of  $Y_t$  and the cross-covariance structure,  $\hat{\lambda}$ s peak around the true changing points with large likelihoods.

structural break at  $t = 0.5T$ ; DGP 2 with a structural break due to the shift of the parameter  $\alpha_{1t}$ , in which  $X_t$  is a stationary process, and  $Y_t$ , as well as the whole bivariate system, is nonstationary, i.e., the true parameter  $\alpha_{1t}$  shifts from 1.2 to 0.3 at  $0.5T$ ; and DGP 4 with an abrupt change followed by a smooth change where the true parameter  $\alpha_{1t}$  has a level shift from 1.2 to  $-0.4\sqrt{2}$  and then smoothly changes. In conclusion, these simulation results show that my estimator performs well in detecting abrupt changes: when abrupt changes exist, the tests of not being an abrupt change are most likely to be rejected at the time spots around the true changing points.

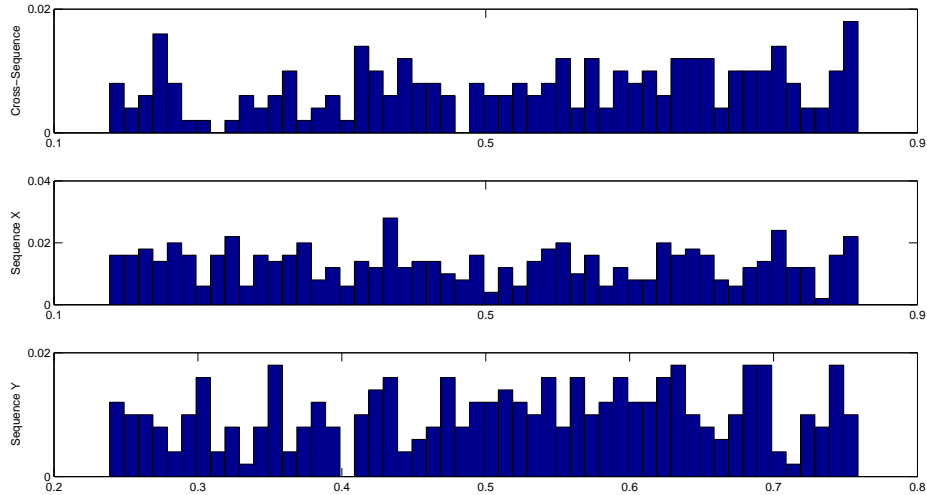


Figure 2.4: DGP 3: Frequency Distribution of  $\hat{\lambda}$

Without an abrupt change in the bivariate process,  $\hat{\lambda}$ s are distributed uniformly and have very small likelihoods.

I also use simulations to investigate the finite sample properties of  $\hat{\lambda}$ , the maximizer of the difference of the left and right spectrum estimators. When there exists at most one abrupt change,  $\hat{\lambda}$  at which the null hypothesis is rejected is defined as the estimator of the changing point. Otherwise no abrupt change is detected. In Table 2.7, I report the sample means and standard de-

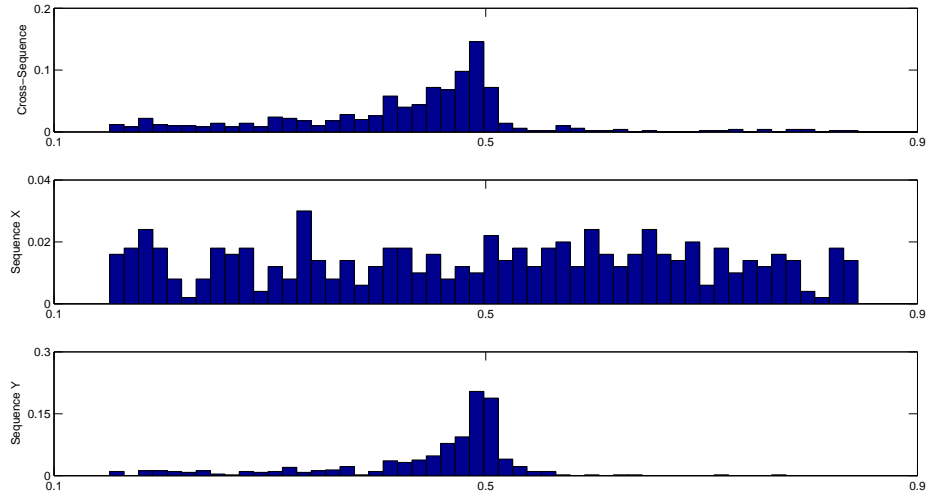


Figure 2.5: DGP 4: Frequency Distribution of  $\hat{\lambda}$

Without an abrupt change in the  $X_t$  process,  $\hat{\lambda}$ s are distributed uniformly and have very small likelihoods. For abrupt changes at  $0.5T$  in the covariance structure of  $Y_t$  and the cross-covariance structure,  $\hat{\lambda}$ s peak around the true changing points with large likelihoods.

viations of  $\hat{\lambda}$  for each data generating processes. The  $\hat{\lambda}$ s are filtered by the 5% significance level in the test. When there is no abrupt change in the sequence or in the system,  $\hat{\lambda}$  has a mean of around 0.5 and a large standard deviation. Also as indicated in the figures of the frequency distribution of  $\hat{\lambda}$ , i.e., Figure 2.1, Figure 2.4, and the plots of the marginal covariance of  $X_t$  in Figure 2.3 and Figure 2.5, without an abrupt change in a process,  $\hat{\lambda}$ s are distributed uniformly and have very small likelihoods of being accepted as the estimates of abrupt changes. In contrast, in Table 2.7, when there is an abrupt change, the sample mean is close to the true changing point, and the sample standard deviation is much smaller, indicating that the estimated abrupt changes are near the true changes. As we can see in Figure 2.2 and the plots of the cross-covariance and the marginal covariance of  $Y_t$  in Figure 2.3 and Figure 2.5,  $\hat{\lambda}$ s peak around the true changing points with large likelihoods.



## 2.8 Empirical application

In this section I use the proposed method to investigate the financial contagion from subprime asset-backed collateralized debt (CDOs) to other markets during the subprime mortgage crisis in 2007 and its aftermath. In this paper, financial contagion is defined in the same way as in Longstaff (2010), i.e., "a significant increase in cross-market linkages after a shock occurs in one market." In this empirical analysis, the changes of dependence are measured by the instabilities of covariance structures between returns in different markets. When a crisis happens, the spillover effect on other markets can be significant in a short time span or be gradual over relatively long time via the correlated-information channel, the liquidity channel, or the risk-premium channel (Longstaff, 2010).<sup>2</sup> In my analysis, I investigate the dynamic patterns of changes in the dependence structures between returns in the subprime market and returns in other financial markets to identify financial contagion.

To measure the returns in the subprime market, the ABX indexes are adopted. The series of indexes contain daily closing values obtained from market dealers for subprime home-equity-based CDOs. The series consist of five indexes based on distinct subprime CDO tranches, rated as AAA, AA, A, BBB, and BBB-, respectively. For example, the AAA index is based on a portfolio of 20 subprime home-equity CDOs with an initial credit rating of AAA. The portfolios are reconstructed every six months. The other four indexes are constructed in a similar way. The riskiness of the portfolios increases with the downgrade

---

<sup>2</sup>Through the first channel, contagion occurs rapidly via the price-recovery process, especially when other markets are more liquid. Under the liquidity mechanism, a shock to the subprime market results in a decrease in the overall liquidity of all financial markets and also affects asset prices. Under the last mechanism, the pessimistic or even panic sentiments of market participants may affect their willingness to bear risks in any market.

of the ratings from AAA to BBB-. I use the returns from the on-the-run indexes. To investigate the contagion effect on other financial markets, I focus on the treasury bond market, the corporate bond market, and the stock market. Specifically, I use daily data of changes in 10-year and 30-year Treasury yields, the changes in Moody's Aaa and Baa credit spreads (obtained by subtracting the 10-year Treasury yield from Aaa and Baa yields), and the S&P 500 return index. The ABX indexes and S&P 500 return index are obtained from Bloomberg, and the bond index data are from the Federal Reserve Board. Using the method developed in this paper, I am able to identify the time spots when the dependence structures change abruptly, which are very likely to be the occurrences of financial contagion.<sup>3</sup> The sample period is from March 2006 to May 2010, containing 1,194 daily observations. The abrupt changing points are tested at the 5% significance level.

Table 2.2, Table 2.3, Table 2.4, and Table 2.5 report the estimated abrupt changes in the covariance structures between the subprime and the security markets, in the marginal covariance structure of each index, in the covariance structures within the subprime market, and in the covariance structures within the security markets, respectively. In each table, each estimated abrupt change is denoted by a date when the change occurs. The number above each date represents the position of the estimated abrupt change in the 1,194 ordered observations. For example, in Table 2.2, the first number 325 represents the first abrupt changing point located at the 325th observation out of 1,194. The superscript denotes the corresponding possible influential event (listed in Table A.1)

---

<sup>3</sup>In a further step, to capture the dynamic patterns of the linkages, a bivariate time-varying vector autoregressive model can be adopted to fit the data, for example, time-varying VAR in the framework of the local stationarity proposed by Dahlhaus (1997). The abrupt changes in the dependence structures are detected in the first step, and the time-varying VAR coefficients measure the smooth changes of the dependence relations. This step is left for future research.

Table 2.2: Abrupt Changes in the Cross-covariance Structures Between the Subprime and the Security Markets

	Aaa	Baa	10Yr	30Yr	S&P
AAA	325 <sup>3</sup> 7/9/2007	320 <sup>3</sup> 6/29/2007	322 <sup>3</sup> 7/3/2007	320 <sup>3</sup> 6/29/2007	326 <sup>3</sup> 7/10/2007
	403 <sup>4</sup> 10/29/2007	403 <sup>4</sup> 10/29/2007	402 <sup>4</sup> 10/26/2007	401 <sup>4</sup> 10/25/2007	401 <sup>4</sup> 10/25/2007
	623 <sup>7</sup> 9/15/2008	627 <sup>7</sup> 9/19/2008	623 <sup>7</sup> 9/15/2008	623 <sup>7</sup> 9/15/2008	623 <sup>7</sup> 9/15/2008
AA	320 <sup>3</sup> 6/29/2007	320 <sup>3</sup> 6/29/2007	322 <sup>3</sup> 7/3/2007	320 <sup>3</sup> 6/29/2007	325 <sup>3</sup> 7/9/2007
	688 12/22/2008	686 12/18/2008	687 12/19/2008	687 12/19/2008	682 12/12/2008
A	304 <sup>2</sup> 6/7/2007	304 <sup>2</sup> 6/7/2007	320 <sup>3</sup> 6/29/2007	320 <sup>3</sup> 6/29/2007	320 <sup>3</sup> 6/29/2007
	320 <sup>3</sup> 6/29/2007	320 <sup>3</sup> 6/29/2007	682 12/12/2008	682 12/12/2008	426 <sup>5</sup> 12/3/2007
	674 12/2/2008	670 11/25/2008			674 12/2/2008
					1027 <sup>8</sup> 5/3/2010
BBB	218 <sup>1</sup> 1/31/2007	223 <sup>1</sup> 2/22/2007	218 <sup>1</sup> 1/31/2007	218 <sup>1</sup> 1/31/2007	223 <sup>1</sup> 2/22/2007
	681 12/11/2008	473 <sup>6</sup> 2/11/2008	681 12/11/2008	681 12/11/2008	679 12/9/2008
		674 12/2/2008			
BBB-	218 <sup>1</sup> 1/31/2007	218 <sup>1</sup> 1/31/2007	212 <sup>1</sup> 1/23/2007	212 <sup>1</sup> 1/23/2007	225 <sup>1</sup> 2/9/2007
	681 12/11/2008	462 <sup>6</sup> 1/25/2008	681 12/11/2008	681 12/11/2008	680 12/10/2008
		671 11/26/2008			

Note: This table shows the estimated abrupt changes in the dependence structures between the returns of the ABX indexes and indexes in the security markets. Each estimated abrupt change is denoted by a date when the change occurs. The number above each date represents the position of the estimated abrupt change in the 1, 194 ordered observations. The superscript denotes the corresponding possible influential event (listed in Table A.1) that is associated with the abrupt change.

Table 2.3: Abrupt Changes in the Marginal Covariance Structures

AAA	343 <sup>3</sup> 8/2/2007	401 <sup>4</sup> 10/25/2007	620 <sup>7</sup> 9/10/2008	
AA	330 <sup>3</sup> 7/16/2007	434 <sup>5</sup> 12/13/2007	462 <sup>6</sup> 1/25/2008	682 12/12/2008
A	320 <sup>3</sup> 6/29/2007	426 <sup>5</sup> 12/3/2007		
BBB	223 <sup>1</sup> 2/22/2007	339 <sup>3</sup> 7/27/2007	462 <sup>6</sup> 1/25/2008	
BBB-	218 <sup>1</sup> 1/31/2007	344 <sup>3</sup> 8/3/2007	462 <sup>6</sup> 1/25/2008	
AAa	304 <sup>2</sup> 6/7/2007	623 <sup>7</sup> 9/15/2008		
Baa	304 <sup>2</sup> 6/7/2007	627 <sup>7</sup> 9/19/2008		
10Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008		
30Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008		
S&P	326 <sup>3</sup> 7/10/2007	623 <sup>7</sup> 9/15/2008	1023 <sup>8</sup> 4/27/2010	

Note: This table reports the estimated abrupt changes in the marginal covariance structures for the returns of each index. Each estimated abrupt change is denoted by a date when the change occurs. The number above each date represents the position of the estimated abrupt change in the 1, 194 ordered observations. The superscript denotes the corresponding possible influential event (listed in Table A.1) that is associated with the abrupt change. The stability of a dependence structure includes not only the covariance stability across returns in different markets but also the marginal covariance stability of the returns for each index.

that is associated with the abrupt change. For example, 2 refers to the event on June 7, 2007 when Bear Stearns & Co informed investors that it was halting redemptions in two of its CDO hedge funds.

Table 2.2 shows the estimated abrupt changes in the dependence structures between the returns of the ABX indexes and indexes in the security markets. The covariance structures between the returns of the AAA index and the returns of

the security indexes all consist of three abrupt changes with the same timing. As indicated in Table A.1, the first set of abrupt changes in late June and early July 2007 may be due to a sequence of shocking news in the subprime market, such as the rating downgrades on a large amount of subprime debt by Standard & Poor's, the failures of Bear Stearns' two CDO hedge funds, and the bankruptcy of American Home Mortgage Investment Corporation (AHMI). My conjecture is that the second set of abrupt changes are caused by the announcements by Merrill Lynch and Citigroup about their huge losses in subprime-related assets at the end of October 2007. The last set of abrupt changes are closely related to the sequence of subprime-related bad news in mid-September 2008, including the nationalization of Fannie Mae and Freddie Mac, the takeover of Merrill Lynch by Bank of America, the bankruptcy of Lehman Brothers, and the consequent downgrades on AIG's credit rating, which was closely connected to the above failing companies by CDS contracts. In the dependence structures of the AA indexes and the securities, there are roughly two sets of abrupt changes. The first set of changing points are almost at the same locations as the first changes in the AAA index and the securities. The second set of abrupt changes are in December 2008, which are not triggered by any specific event but can be considered as an accumulative result of the U.S. housing bubble burst and the global financial crisis. The abrupt changes between the A index and securities in the corporate bond market, the Treasury bond market, and the stock market are detected at various locations. For the pair of the A index and Moody's Aaa and Baa, in addition to the two abrupt changes as in the AA index and the corporate bond spreads, the first set of changing points are around June 7 2007, when Bear Stearns informed investors in two of its CDO hedge funds (the High-Grade Structured Credit Strategies Enhanced Leverage Fund and

the High-Grade Structured Credit Fund) that it was halting redemptions. The abrupt changes in the dependence structures between the A index and the Treasury bonds have the same locations as the changes between the AA index and the Treasury bonds. For the linkage between the A index and the S&P 500 index, besides the two changes detected at the same locations as those between the AA index and the S&P index, there are two additional changing points detected at the end of 2007 and in May 2010, respectively. The abrupt change in December 2007 may be due to the creation of a Term Auction Facility (TAF) by the Federal Reserve Board to address pressures in the short-term funding market.<sup>4</sup> The other change in May 2010 may be triggered by the announcement of Standard & Poor's downgrade on the debt ratings of Greece and Portugal. For the two indexes based the most risky and least liquid subprime CDO tranches, the BBB and BBB- indexes, the abrupt changes are in similar patterns. They both have the same abrupt changes in the dependence structures with indexes in the security markets, with the first abrupt change in January/February 2007, and the second one in December 2008. The first set of changes are likely the consequence of subprime mortgage firms' bankruptcy and HSBC's warning of its huge losses in the subprime market. The second set of changes are also closely related to the US housing bubble burst and the global financial crisis. Moreover, the two subprime indexes both have an additional change with Moody's Baa in early 2008. This is highly related to the continuing housing market plummet, especially after the announcement by the National Association of Realtors (NAR) that 2007 had the largest drop in existing home sales in 25 years and "the first price decline in many, many years and possibly going back to the Great Depression."

---

<sup>4</sup>In TAF, fixed amounts of term funds will be auctioned to depository institutions against a wide variety of collateral.

Table 2.4: Abrupt Changes in the Cross-covariance Structures in the Sub-prime Market

AAA/AA	330 <sup>3</sup> 7/16/2007	434 <sup>5</sup> 12/13/2007	687 12/19/2008
AAA/A	329 <sup>3</sup> 7/13/2007	434 <sup>5</sup> 12/13/2007	687 12/19/2008
AAA/BBB	325 <sup>3</sup> 7/9/2007	462 <sup>6</sup> 1/25/2008	687 12/19/2008
AAA/BBB-	325 <sup>3</sup> 7/9/2007	462 <sup>6</sup> 1/25/2008	681 12/11/2008
AA/A	325 <sup>3</sup> 7/9/2007	426 <sup>5</sup> 12/3/2007	682 12/12/2008
AA/BBB	325 <sup>3</sup> 7/9/2007	462 <sup>6</sup> 1/25/2008	674 12/2/2008
AA/BBB-	325 <sup>3</sup> 7/9/2007	415 <sup>4,5</sup> 11/15/2007	462 <sup>6</sup> 1/25/2008
A/BBB	320 <sup>3</sup> 6/29/2007	426 <sup>5</sup> 12/3/2007	462 <sup>6</sup> 1/25/2008
A/BBB-	320 <sup>3</sup> 6/29/2007	426 <sup>5</sup> 12/3/2007	462 <sup>6</sup> 1/25/2008
BBB/BBB-	224 <sup>1</sup> 2/8/2007	342 <sup>3</sup> 8/1/2007	462 <sup>6</sup> 1/25/2008

Note: This table reports the estimated abrupt changes of the dependence structure across the indexes within the subprime market. Each estimated abrupt change is denoted by a date when the change occurs. The number above each date represents the position of the estimated abrupt change in the 1, 194 ordered observations. The superscript denotes the corresponding possible influential event (listed in Table A.1) that is associated with the abrupt change. Most of these abrupt changes in the cross-covariance structures are coincident with the abrupt changes in the marginal covariance structures of the more risky asset returns.

In Table 2.3, I report the estimated abrupt changes in the marginal covariance structures for the returns of each index. The stability of a dependence structure includes not only the covariance stability across returns in different markets but also the marginal covariance stability of the returns for each index. The marginal covariance structure measures the stationarity of fluctuation over time for each time series. The abrupt changes or smooth changes in a covariance structure are due to exogenous shocks or endogenous evolution over a long time period. More importantly, the stability of the cross-covariance and thus the dependence structure relies on the stability of the marginal covariance structures of both return sequences. Comparing Table 2.2 and Table 2.3, we can see that most of the abrupt changes in the cross-covariance structures match the changes in at least one of the marginal covariance structures. It also happens frequently that the abrupt changes occur simultaneously in cross-covariance and both of the two corresponding marginal covariances. For instance, the first two sets of abrupt changes between the AAA index and the bond market indexes coincide with abrupt changes in the marginal covariance structure of the AAA index itself. However, the third set of abrupt changes between AAA and the bond markets may be under some common effects to both of the markets. In other words, these simultaneous abrupt changes can be the result of common economic factors affecting multiple markets or rapid spillovers from one market to another. On the other hand, changes in a marginal covariance structure do not necessarily lead to changes in its corresponding cross-covariance when the two return sequences are barely dependent. For example, the abrupt changes in the marginal covariance structure of the AA index in late 2007 and early 2008 have no influences on the cross-covariance structure. Furthermore, some of the changes in a cross-covariance are exclusively due to changes in the correlation,



Table 2.5: Abrupt Changes in the Cross-covariance Structures in the Security Markets

Aaa/AAb	304 <sup>2</sup> 6/7/2007	626 <sup>7</sup> 9/18/2008	
Aaa/10Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008	
Aaa/30Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008	
Aaa/S&P	304 <sup>2</sup> 6/7/2007	623 <sup>7</sup> 9/15/2008	1023 <sup>8</sup> 4/27/2010
Baa/10Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008	
Baa/30Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008	
Baa/S&P	304 <sup>2</sup> 6/7/2007	623 <sup>7</sup> 9/15/2008	1023 <sup>8</sup> 4/27/2010
10Yr/30Yr	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008	
10Yr/S&P	304 <sup>2</sup> 6/7/2007	623 <sup>7</sup> 9/15/2008	1023 <sup>8</sup> 4/27/2010
30Yr/S&P	304 <sup>2</sup> 6/7/2007	622 <sup>7</sup> 9/12/2008	1023 <sup>8</sup> 4/27/2010

Note: This table reports the estimated abrupt changes of the dependence structure across the indexes within the security markets. Each estimated abrupt change is denoted by a date when the change occurs. The number above each date represents the position of the estimated abrupt change in the 1, 194 ordered observations. The superscript denotes the corresponding possible influential event (listed in Table A.1) that is associated with the abrupt change.

rather than due to the changes in either corresponding marginal covariance, e.g., the pervasive abrupt change in the dependence relations at the end of 2008, probably as a result of the U.S. housing bubble burst and the global financial crisis.

I also report the estimated abrupt changes in the dependence structures across the indexes within the subprime market in Table 2.4 and within the security markets in Table 2.5. For the subprime market, all of the cross-covariance structures have three abrupt changes. Most of these abrupt changes in the cross-covariance structures coincide with the abrupt changes in the marginal covariance structures of the more risky asset returns. For example, the first two out of the three abrupt changes in the covariance structure between the AAA and AA indexes are closely related to the first two changes in the marginal covariance structure of the AA index. Similarly, the first two abrupt changes in the pairs of A/AAA and A/AA are dominated by the changes in the marginal covariance structure of the A index. The first two abrupt changes in BBB/AAA, BBB/AA, and BBB/A can be attributed to the last two abrupt changes in the marginal covariance structure in BBB. Also, the first two abrupt changes in BBB-/AAA, BBB-/AA, BBB-/A, and BBB-/BBB occur almost at the same time as the last two abrupt changes in the marginal covariance structure in BBB-. Furthermore, the third abrupt change in the pairs of AA/AAA, A/AAA, BBB/AAA, BBB-/AAA, A/AA, and BBB/AA are in accordance with the occurrence of the global financial crisis at the end of 2008. For the bond market, since all bond indexes have the abrupt changes simultaneously, it is natural that the cross-covariance structures within the bond market have the same dynamic patterns, i.e., having abrupt changes around June 7, 2007 and September 15, 2008. With respect to the dependence between the bond market and the stock market, the abrupt change in the S&P 500 index on April 27, 2010 affects all of the dependence structures.

Taken together, the tables show that the covariance structures of ABX indexes, in particular the indexes with lower credit ratings, contain fewer abrupt changes during the crises after 2007. Many negative shocking events that trig-

gered abrupt changes in the covariance structures between AAA or AA indexes does not lead to simultaneous fundamental abrupt changes in the lower-rated ABX indexes. Furthermore, the changes in the marginal covariance structures are less likely to lead to changes in the dependence structures than before in the lower-rated indexes. These are probably because that the subprime CDO insurance declined precipitously once the crisis began in late 2007 and the market liquidity dried up quickly. As a result, the illiquidity shocks fail to reflect any fundamental structural changes in the covariance structures.

Interestingly, in the cross covariance structures between the pairs of index returns, the marginal covariance structures of returns, many of the abrupt changes coincide. On the one hand, such as shown in Figure A.1 to Figure A.15, the time lines of the abrupt changes in covariance structures between subprime markets and similar major financial markets are almost the same, except a slight difference in the exact dates. For example, in Figure A.2, the abrupt changes follow the same patterns between the ABX\_AAA index and the two indexes in Treasury bonds. On the other hand, the abrupt changes detected between pairs of return indexes are also similar cross markets. The most significant evidence lies in Figure A.4 to Figure A.15, the abrupt changes identified at the end of 2008, among dependence structures between subprime markets, and the other financial markets except ABX\_AAA, although there are no corresponding influential shocking events. The synchrony of the changes is probably driven by some common economic factors affected some specific markets or the entire financial sector.

## 2.9 Conclusion

In financial markets, economic relationships may change abruptly as the result of rapid market reactions to exogenous shocks, or change gradually over a long time span due to various activities and different responses of multiple market participants. For example, the studies on financial contagion concentrate on changes of dependence structures among economies, industries or institutions. These changes in dependence can be measured by the instabilities of the covariance structure between two asset returns. In this paper, a spectrum-based estimator is proposed to detect abrupt changes in covariance structures. This paper brings together and improves upon two strands of literature by providing a spectrum-based estimator accommodating the scenario where a time series has a nonstationary covariance structure due to either abrupt or smooth changes or both. Compared to the structural break literature which mainly considers structural changes as discrete level shifts in an observation period, my method is more general by allowing occasional breaks to occur in a smooth change circumstance approximated by locally stationary processes, thus subsuming level shifts as a special case. This method also extends the strand of literature on smooth changes approximated by local stationarity by relaxing the assumption of continuity and introducing abrupt changes. Simulation results show that the estimator performs well in finite samples for data generating processes with either abrupt or smooth changes and processes with both changes. I also apply the estimator to index returns data to empirically detect abrupt changes in the interdependence relations across subprime mortgage, stock, and bond markets in the U.S. during the 2007 subprime crisis and the 2008 global financial crisis. The empirical results show that the covariance structure does have abrupt changes

right after some bad events, such as the sequence of subprime-related bad news in mid-September 2008, including the nationalization of Fannie Mae and Freddie Mac, the takeover of Merrill Lynch by Bank of America, the bankruptcy of Lehman Brothers, and the consequent downgrade ratings on AIG's credit. On the other hand, abrupt changes are not necessarily triggered by specific events. For example, the abrupt change in December 2008 can be considered as an accumulative result of the U.S. housing bubble burst and the global financial crisis. What's more, most of the changes in the dependence structures are closely related to the changes in the marginal covariance structures of the returns. However, not all of the changes in the marginal covariance structures lead to changes in the cross-covariance structures.

CHAPTER 3

**POST-CRISIS GLOBAL LIQUIDITY AND FINANCIAL SPILLOVER:  
FROM U.S. TO EMERGING MARKETS**

### **3.1 Funding Liquidity and Market Liquidity**

Literature defines liquidity in various ways. Some scholars regard liquidity as the availability of funds can be obtained from financial intermediaries. In this definition, funds are assets or arrangements that can be converted into a medium of exchange. Others regard liquidity as the ease of convertibility of assets. For example, stocks of large firms and Treasuries bonds are usually more liquid than corporate bonds of relatively small firms. Brunnermeyer and Pedersen (2009), among others, propose a model to illustrate the interaction between funding liquidity and market liquidity. When funding liquidity is tight, traders (hedge funds, dealers, or investment banks) become hesitate in taking on positions, especially capital intensive positions in high-margin securities. So the market liquidity is lowered with higher volatility. Under certain conditions, low future market liquidity increases the risk of financing a trade, thus further dragging down funding liquidity and increasing margins. The right measure of liquidity depends on the precise meaning of liquidity and on the specific question.

The funding liquidity spillover literature can be categorized into domestic liquidity spillover and global liquidity spillover. This paper belongs to the latter. In the domestic liquidity literature, Frank, Gonzalez-Hermosillo and Hesse (2008) discuss the liquidity spillover across five financial markets in U.S. during the 2007 subprime crisis. They estimate correlations by Engle (2002) Dynamic

Conditional Correlation model (DCC) and concluded that the linkage between market and funding liquidity increased sharply during the crisis, and that bank solvency became important. Longstaff (2010) investigates financial contagion from the subprime market to the corporate bond, the Treasury bond, and the stock markets. Using the estimation results of Vector Autoregression, he reaches the conclusion that the funding liquidity was a major factor in the transmission of the contagion during the crisis.

Among the global liquidity spillover literature, Frank and Hesse (2009) focus the U.S. liquidity spillover to emerging markets during the global financial crisis. In their paper, the correlations between liquidity and stock, bond and credit markets in emerging economies during the crisis are estimated by DC-C. Fratzscher, Duca, and Straub (2012) analyze the global liquidity spillover caused by the Federal Reserve's unconventional monetary policy measures since 2007. They find that the first round of quantitative easing, rather than the second round, was highly effective in lowering yields and raising equity market in emerging economies. This paper is the one that is most related to my research, while has a different conclusion.

Klye (1985) distinguishes market liquidity literature among three sub-categories: the bid-ask spread, measuring the loss of traders on selling one unit of an asset and buying it back right away; market depth, measuring amounts that traders can selling and buy on current bid or ask price; market resiliency, measuring the time needed for prices that have temporarily fallen to bounce back. In the empirical literature, Amihud (2002) constructs an illiquidity measure based on Klye (1985) for the U.S. equity market and it has been extended to the corporate bond market by Han and Zhou (2008). Other measures of market

liquidity include but are not limited to Roll (1984) measure, Imputed roundtrip cost (IRC) by Feldhutter (?), Turnover, and Zero trading days. Among literature discussing global market liquidity spillover effects, Lee (2011) investigate the spillover of illiquidity and liquidity risk from U.S. to the rest of the world based on the liquidity-adjusted capital asset pricing model of Acharya and Pedersen (2005). Karolyi, Lee, and van Dijk (2012) examine how commonality in liquidity varies across countries and over time in ways related to supply determinants (funding liquidity of financial intermediaries) and demand determinants (correlated trading behavior of international and institutional investors, incentives to trade individual securities, and investor sentiment) of liquidity. Chordia, Sarkar, and Subrahmanyam (2005, 2011) focus on market liquidity spillover effects between the U.S. stock and bond markets.

## **3.2 Data**

In this paper, I use the TED spread in the U.S. and its analogues in the emerging economies to measure the funding liquidity. To capture the changes in the linkages among financial markets between U.S. and the emerging economies, I mainly focus on the bond markets and the equity markets. The key linkages are summarized via the financial variables based on U.S. Dollar-denominated bond and equity indexes.



### 3.2.1 Measure of funding liquidity

As mentioned in previous sections, the TED spread is the difference between the LIBOR rate and Treasury bill rate. Usually, the LIBOR rate and U.S. Treasury bill rate are both considered risk-free, and thus the spread between them is limited and stable. However, in times of financial turmoil, banks charge higher interest for unsecured loans, which increases the LIBOR rate. In the mean time, due to “fly to quality” and “fly to liquidity”, U.S. Treasury bills become more attractive and the rates are pushed down. For both reasons, as indicated in Figure 3.1 the TED spread widens during crises and narrows down in tranquil periods. In this paper, the TED spread is defined as the difference between 3-month LIBOR rate and Treasury bill rate.

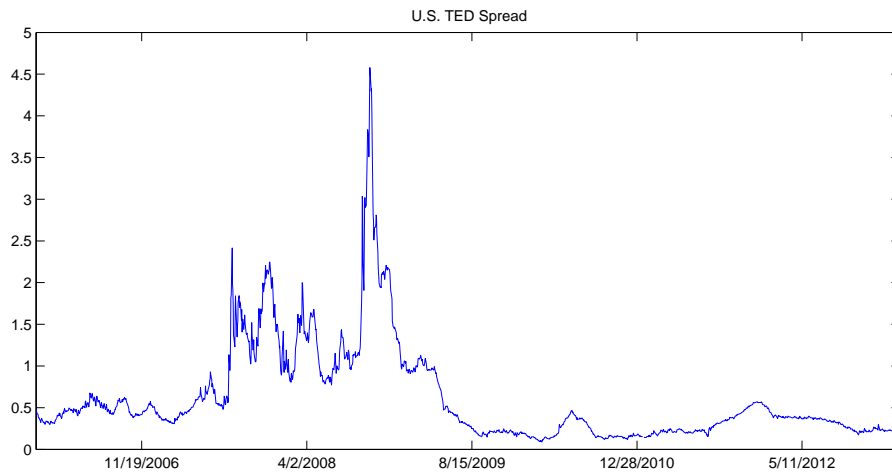


Figure 3.1: U.S. TED Spread

The U.S. TED spread is frequently used by researchers and market practitioners to measure the funding liquidity in the U.S. markets.

The U.S. TED spread is frequently used by researchers and market practitioners to measure the funding liquidity in the U.S. markets. To make it consistent, I construct the analogues of the U.S. TED spread for the emerging e-

conomies (as shown in Figure 3.2). Admittedly, there is controversy about risk-free rates in emerging markets, since emerging government bonds cannot be considered riskless. Nevertheless, BRICS, as the largest, fastest-growing, and most influent emerging economies around the world, are relatively stable and less likely to default their government bonds during recent financial crises. In these economies, government bonds have the least risk among investment instruments. Also, in the case of financial turmoil, it is more realistic and easier for investors to purchase the local government bonds instead of holding the U.S. Treasury bonds in their portfolios to avoid risks. For the above reasons, to a certain degree, the TED spread constructed using the short-term treasury rate as a risk-free rate still reflects the changes in funding liquidity in these emerging markets. Next, the details of the TED spreads in emerging economies are discussed.

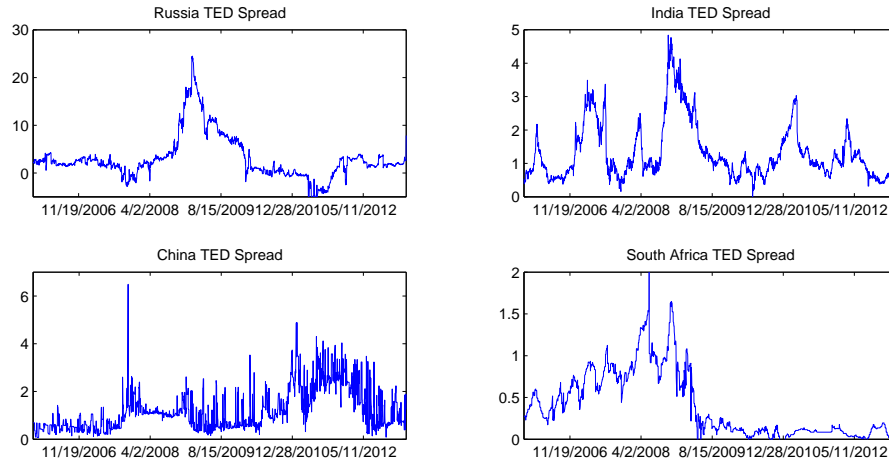


Figure 3.2: TED Spreads for Emerging Markets

The analogues of the U.S. TED spread for the emerging economies are constructed to measure the funding liquidity of emerging markets.

In the Russian financial markets, the TED spread is defined as the difference between 31 to 90-days Moscow Interbank Offered Rate (MIBOR) and GKO

market rate. GKO bonds are short-term zero coupon Russian Government Treasury Bills issued by the Russia Finance Ministry. GKO bonds defaulted in 1998 Russian financial crisis, which was the most significant financial crisis in post-Soviet Russia. After 1998, a new series of the government bonds was issued. Figure 4 depicts the TED spread, with the peak in early 2009 and with a similar shape as the U.S. TED spread.

Similarly to the Russian markets, I construct the TED spread in the Indian markets as the difference between 3-month Mumbai Interbank Offered Rate (another MIBOR) and 91 days Treasury Bills. The Indian TED spread has a very similar pattern as the U.S. TED spread, both with global peak in early 2009, while it fluctuates more dramatically with multiple local peaks in 2006, 2011 and 2012.

For the Chinese markets, instead of Treasury bill rate as the risk-free rate, the central bank bill rate is used, because the Chinese Treasury bill markets are rather immature compared to the developed financial markets. Furthermore, there are two systems of the interbank offered rate, i.e., China Interbank Offered Rate (Chibor) and Shanghai Interbank Offered Rate (Shibor). Although called offered rate, Chibor, introduced in 1996, is based on the real trading interest rates among banks in China. On the other hand, Shibor, introduced in October 2006, is the daily rate based on the interest rates at which banks offer to lend unsecured funds to other banks in the Shanghai money market. In this paper, the Chinese TED spread is constructed as the spread between 3-month Chibor rate and 3-month central bank bill rate. In South African financial market, the TED spread is the difference between the 3-month Johannesburg Interbank Agreed Rate (JIBAR) and the 91-days Treasury bill rate. The SA TED spread also

behaves similarly to the U.S. TED, while it is more volatile before its peak in 2008 and early 2009. After that, the spread becomes very flat.

In 1999, Central Bank of Brazil substituted the interbank basic interest rate and offered rate by the target SELIC rate, which is the overnight lending rate determined by the Central Bank of Brazil's Monetary Policy Committee (COPOM). Therefore, there is no relevant proxy available to construct a TED spread as the measure of funding liquidity in the Brazilian markets.

### **3.2.2 Financial Variables in Bond and Equity Markets**

In the investigation of the linkages between the U.S. and the emerging markets, I mainly focus on the bond markets (Figure 3.3) and the equity markets (Figure 3.4). In the U.S. markets, I choose Barclays Capital U.S. Aggregate Bond Index (AGG) and S&P 500 Composite Index. AGG covers U.S. Dollar-denominated, investment-grade, fixed-rate, taxable bond market of SEC-registered securities. It includes bonds from the Treasury, government-related, corporate, MBS, ABS and CMBS sectors.

In the emerging markets, I use J.P. Morgan Emerging Market Bond Index Global (EMBI Global) as the benchmarks of the bond markets in BRICS. The EMBI Global includes the U.S. Dollar-denominated Brady bonds, Eurobonds, traded loans, and local market debt instruments issued by sovereign and quasi-sovereign entities. Besides, the S&P/IFCI is adopted as the measures for the equity markets. The index members have a minimum float-adjusted market cap of US\$ 200 million, a minimum annual dollar value traded of US\$ 100 million, and have less than four no-trades in the two consecutive quarters prior to

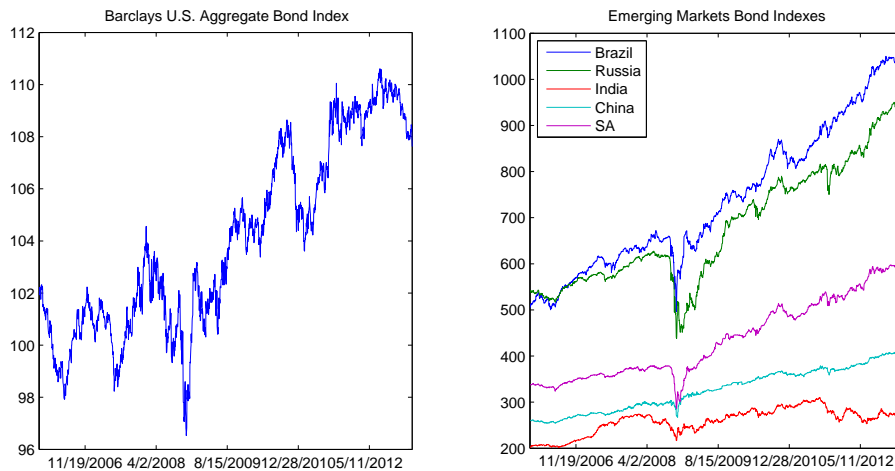


Figure 3.3: Bond Indexes for U.S. and Emerging Economies

Barclays Capital U.S. Aggregate Bond Index (AGG) and J.P. Morgan Emerging Market Bond Index Global (EMBI Global) are used as the benchmarks of the bond markets in U.S. and BRICS, respectively.

reconstitution.

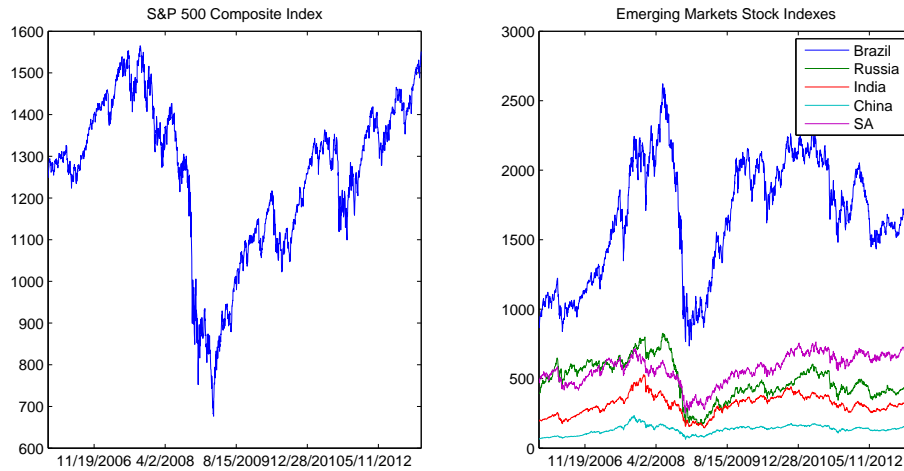


Figure 3.4: Stock Indexes for U.S. and Emerging Economies

S&P 500 Composite Index and S&P/IFCI are used as the benchmarks of the stock markets in U.S. and BRICS, respectively.

To summarize, the interbank offered rates, the Treasury bill rates, the central bank bill rates, the bond indexes, and the stock indexes are collected from Datas-

tream, Federal Reserve Board, Central Bank of Russia, and South Africa Reserve Bank. The dataset contains daily data from January 3rd, 2006 to February 28th, 2013, with 1758 observations for each time series. To apply methodology introduced in the next section, the change/return data are constructed accordingly. The effective sample is July 14th 2006 to August 17th 2012.

### **3.3 Methodology**

This paper discusses the liquidity spillover during the period of the financial turmoil and the intensive implementation of monetary policies. The data of the U.S. markets and the emerging markets, as well as their linkages are likely to contain multiple structural changes. These changes can be abrupt or smooth. For example, at the beginning of a crisis, investors may suddenly change their investment behaviors, such as seeking to sell riskier assets and instead purchase safe assets such as Treasury bonds and gold; banks may suddenly stop or slow their lending activities. These abrupt changes in the behaviors cause abrupt changes in time series of returns and other stochastic processes used to describe the market conditions. These changes are prone to be but not necessarily triggered by negative shocking events. In contrast, during the recovery from a crisis, the structures may change smoothly due to various actions of multiple market participants at different points in time, such as fiscal stimulus, monetary expansion, bankruptcies and acquisitions, and adjustments in investment strategies. Therefore, the conventional econometric models with the assumption of data stationarity are not applicable and otherwise will lead to unreliable statistical inferences and predictions.

The two-step method used is able to capture the liquidity spillover effects with the nonstationary data due to structure changes. In the first step, I use a spectrum-based nonparametric method to detect abrupt changes. In the second step, I fit a time-varying Vector Autoregression (VAR) to the data in each segment split by the detected abrupt changing points. This method has the following features: First, no justification of the types of changes is needed. This method detects the abrupt changes in the first step and captures the smooth changes in the second step. Second, both of the abrupt changes and smooth changes can be captured endogenously if they exist, and no additional assumptions on locations of the changes are required.

### **3.3.1 Detection of abrupt changes**

The spectrum-based nonparametric method is proposed by Zheng (2012). This method can detect abrupt changes both in a stochastic process and in the dependence structure a pair of stochastic processes. In her paper, the dependence structure is measured by cross-covariance structure, which consists of contemporary cross-covariance and all lag orders of cross auto-covariance. Therefore, this cross-covariance structure comprehensively captures both co-movement and all possible lead-lag effects. This method focuses on detecting abrupt changes in this covariance structure via its corresponding cross-spectral density. In this method, the stochastic processes are allowed to have multiple abrupt changes with the number and locations unknown. Except of the abrupt changes, the processes change smoothly as local stationary processes, or even keep stationary. Thus detecting the abrupt changes is equivalent to locating step discontinuities in the time-varying cross-spectral density function. The estimates

of the true changing points then can be implemented based on a comparison of the left and right limits of the spectral density function at each potential time spot. This method can be applied to detect abrupt changes in a stochastic process as a special case.

### 3.3.2 Time-varying Vector Autoregression

As mentioned above, once the processes are segmented by the detected abrupt changes, the sub-processes are assumed to be locally stationary or stationary. Similar to Dauhllhaus (1997), I fit a time-varying VAR model of order  $p$  to the data in each segment

$$Y_t = C(t) + \sum_{l=1}^p A_l(t) Y_{t-l} + U_t,$$

where  $Y_t = [EMTED_t, EMBI_t, IFCI_t, TED_t, AGG_t, SPX_t]'$  with  $EMTED_t$ ,  $EMBI_t$ , and  $IFCI_t$  as the measure of liquidity, the bond market index, and the stock market index for the five emerging economies, respectively;  $TED_t$ ,  $AGG_t$ , and  $SPX_t$  as the measure of liquidity, the bond market index and the stock market index for the U.S. markets. Furthermore, the coefficient  $A_l(t)$  is an  $6 \times 6$  matrix with its element  $a_l^{i,j}(t)$  modeled as polynomials with different orders:

$$a_l^{i,j}(t) = \sum_{k=0}^{K_l} b_l^{i,j} \left( \frac{t}{T} \right)^k, l = 1, \dots, p.$$

Since this method focuses on the changes in the second order moment, (including variance, cross covariance, auto-covariance, and cross auto-covariance), the intercepts  $C(t)$  and the variance-covariance matrix of  $U_t$ ,  $\Sigma$  are both assumed to be constant. The model with time-varying intercepts and  $\Sigma_t$  is left for future research. Then the model orders  $p, K_1, \dots, K_p$  are chosen by minimizing the Akaike



Information Criterion (AIC):

$$AIC(p, K_1, \dots, K_p) = \log \left| \widehat{\Sigma}(p, K_1, \dots, K_p) \right| + \frac{2n \left( n \left( p + \sum_{l=1}^p K_l \right) + 1 \right)}{T}$$

with  $n = 6$  and  $\widehat{\Sigma}$  the estimate of  $\Sigma$ .

## 3.4 Empirical Results

### 3.4.1 U.S. versus Russia

Table 3.1 reports the abrupt changes between the Russian and U.S. markets. The top part of the table reports the detected abrupt changes among the financial variables in the Russian markets, including the abrupt changes in the Russian TED spread (RuTED), the EMBI Global Russia index (EMBI), and the IFCI Russia index (IFCI), as well as the abrupt changes in the dependence structures in the pair of the TED spread and the EMBI Global index (RuTED\*EMBI), the pair of the TED spread and the IFCI index (RuTED\*IFCI), and the pair of the EMBI Global index and the IFCI index (EMBI\*IFCI). The middle part of the table reports the detected abrupt changes among the financial variables in the U.S. market. Similar to the top part, this part includes the detected abrupt changes in the time series, i.e., the U.S. TED spread (TEDSP), the U.S. bond index (AGG), and the S&P stock index (SPX). It also contains the detected abrupt changes in the pairwise dependence relationships, i.e., TEDSP\*AGG, TEDSP\*SPX, and AGG\*SPX. The bottom part of the table shows the abrupt changes detected in the pairwise dependence relationships between the Russian and the U.S. markets.

To some degree, the findings in the table suggest the pattern of the financial

Table 3.1: Abrupt Changes Detected among the financial variables in Russia-U.S.

Abrupt Changes Detected among the financial variables in Russia						
RuTED		10/20/2008	2/11/2010			2/1/2012
EMBI		10/10/2008		4/22/2010		9/19/2011
IFCI		9/16/2008		7/7/2010	8/3/2011	12/13/2011
RuTED*EMBI		10/10/2008		6/7/2010		9/19/2011
RuTED*IFCI		10/6/2008	2/11/2010	6/3/2010	8/8/2011	2/1/2012
EMBI*IFCI		9/16/2008		4/27/2010	8/4/2011	11/4/2011
Abrupt Changes Detected among the financial variables in U.S.						
TEDSP		7/24/2007	10/21/2008	5/11/2010	7/21/2011	8/26/2011
AGG	6/7/2007		11/26/2008		1/7/2011	11/2/2011
SPX	6/7/2007	7/24/2007	9/15/2008	4/27/2010		8/2/2011 12/1/2011
TEDSP*AGG	6/7/2007		11/5/2008	5/11/2010	1/7/2011	8/25/2011
TEDSP*SPX		7/24/2007	9/15/2008	6/7/2010		8/26/2011
AGG*SPX	6/7/2007		9/15/2008	4/27/2010		7/29/2011 11/14/2011
Abrupt Changes Detected among the financial variables between Russia and U.S.						
RuTED*TEDSP	6/26/2007	7/24/2007	9/15/2008	2/11/2010		7/21/2011 9/29/2011
RuTED*AGG			9/29/2008			7/29/2011 2/1/2012
RuTED*SPX	7/6/2007		9/29/2008	2/11/2010	6/7/2010	8/4/2011 2/1/2012
EMBI*TEDSP	6/7/2007		9/15/2008		5/28/2010	7/26/2011 10/13/2011
EMBI*AGG	6/7/2007		9/15/2008		4/22/2010	7/28/2011 11/2/2011
EMBI*SPX	6/7/2007		9/15/2008		4/27/2010	8/4/2011 12/1/2011
IFCI*TEDSP			9/15/2008		5/28/2010	7/21/2011 9/23/2011
IFCI*AGG			9/15/2008		7/7/2010	7/29/2011 11/4/2011
IFCI*SPX			9/15/2008		4/27/2010	7/29/2011 12/1/2011

Note: This table reports the abrupt changes between the Russian and U.S. markets. The top part of the table reports the detected abrupt changes among the financial variables in the Russian markets, the middle part reports the changes in the U.S. market, and the bottom part shows the abrupt changes detected in the pairwise dependence relationships between the Russian and the U.S. markets.

crisis spilling over from the U.S. to the Russian markets. It can be seen in the table that before September 2008, there was no abrupt changes detected in the Russian markets. However, there were two abrupt changes in the U.S. markets. The first detected abrupt change, on June 7th, 2007 corresponded to the day when Bear Stearns & Co. informed investors that the big losses in two of its CDO hedge funds. The effect of this shocking event was limited in the U.S.

bond and equity markets.

The second abrupt change in the stock index detected on July 24th, 2007 was more likely to be an endogenous change as a cumulative result of negative shocks than to be triggered by specific shocking news. In that week, the overall U.S. stock market just experienced the “buying climax” and started to collapse before the weekend. This change limited itself in the U.S. stock market and did not trigger an abrupt change in the bond index. Overall, these two abrupt changes affected the dependence structures between the markets in the two economies.

The third influent abrupt change of the stock index was on September 15th, 2008, when Lehman Brother filed for bankruptcy protection. Indeed, this changing point can be considered as the result of a series of negative shocking news, also including that Merrill Lynch was sold to Bank of America on September 14th, and Moody’s and Standard and Poor’s downgraded ratings on AIC’s credit on September 16th. Different from the previous shocking news with the impact merely in the U.S. market, this shock triggered abrupt changes almost simultaneously in the U.S. and the Russian stock markets. Furthermore, the shock also spread to the bond markets in both economies and triggered abrupt changes very soon. Since September 2008, most of abrupt changes were synchronous. This interesting phenomenon also can be observed between the U.S. markets and other major emerging markets that will be discussed below. Therefore, September 2008 can be considered as the beginning of the global financial crisis. Last but not least, the abrupt change in the U.S. bond index detected on November 26th may be related to the formal launching of Federal Reserve QE 1 to repair the functioning of the financial markets.

The group of abrupt changes in the U.S. and Russian markets between April and July 2010 were closely related to the European sovereign debt crisis, as a part of the global financial crisis. In particular, on April 27th, Standard and Poor's downgraded Greece's sovereign credit rating to junk after the activation of a €45 billion EU-IMF bailout, which triggered the decline of the stock market worldwide and of the Euro's value, and furthered the crisis.

The last group of major abrupt changes were mainly due to the crash of global stock market during late July and early August, 2011. The dramatical plummet of stock prices was triggered by Standard and Poor's downgrade of America's credit rating from AAA to AA+, as well as a cumulative result of fears of contagion of the European sovereign debt crisis to Spain and Italy, and concerns over France's credit rating. In the rest of year, severe volatility of the stock markets continued and cumulated to other structural changes.

Table 3.2 shows the estimation results of VAR in six subsamples segmented by some of the detected abrupt changes discussed above.<sup>1</sup> The optimal  $p$   $K_1, \dots, K_p$  are determined by the AIC criterion. The results show that the data keep stationary in the subsamples as  $K_l = 0$ . In the two segments before mid-September 2008, when the U.S. subprime crisis gradually spread to the general U.S. financial markets, i.e., the period between July 14th 2006 and June 6th 2007, and the period between July 24th 2007 and September 12th 2008, SPX had a one-lag-ahead positive lead effect on both the bond market and equity market in Russia. Meanwhile, AGG also had one-lag-ahead positive lead effect on the Russian bond market. In October 2008 to February 2010, the period of the beginning of the global financial crisis, U.S. Federal Reserve launched the first round

---

<sup>1</sup>I only report the results of segments with over 100 observations for each time series and equations of the emerging economies.

Table 3.2: VAR(P) Coefficients for Russia-U.S.

7/14/2006-6/6/2007		AR(1)						R2
Days:221	C	RuTED	EMBI	IFCI	TEDSP	AGG	SPX	
RuTED	0.0025	(0.2708)***	(2.2247)	(0.1680)	(0.0559)	5.0875	2.5183*	0.0953
EMBI	0.0004***	0.0016	(0.1751)**	0.0068	(0.0041)	0.4882***	0.0582***	0.1359
IFCI	(0.0000)	0.0017	(0.4681)	(0.0321)	0.0391	(0.4450)	0.6772***	0.0838
7/24/2007-9/12/2008		AR(1)						R2
Days:282	C	RuTED	EMBI	IFCI	TEDSP	AGG	SPX	
RuTED	0.0021	(0.3578)***	(0.1821)	1.1950**	(0.1630)	(0.7780)	0.4551	0.1554
EMBI	0.0002	(0.0011)	0.0611	0.0087	0.0004	0.2041***	0.0246**	0.1158
IFCI	(0.0006)	(0.0035)	(1.1324)*	0.0046	(0.0175)	0.2988	0.4174***	0.1023
10/21/2008-2/10/2010		AR(1)						R2
Days:321	C	RuTED	EMBI	IFCI	TEDSP	AGG	SPX	
RuTED	0.0198	(0.2036)***	(2.3113)	(0.8787)	(0.4525)	(0.8955)	1.6738	0.0598
EMBI	0.0009*	(0.0015)	0.1948***	0.0041	0.0022	0.0851	0.1093***	0.1328
IFCI	0.0023	(0.0035)	0.1917	(0.0058)	0.0071	(0.7558)	0.4248***	0.0749
7/7/2010-1/6/2011		AR(1)						R2
Days:123	C	RuTED	EMBI	IFCI	TEDSP	AGG	SPX	
RuTED	(0.0018)	(0.3412)***	1.4352	(0.5828)	(0.0203)	1.9419	(0.0758)	0.1456
EMBI	0.0004	(0.0043)	0.2759**	(0.0557)	0.0093*	0.4412***	0.1000**	0.2622
IFCI	0.0021*	(0.0121)	0.0444	0.0754	0.0426**	0.6683	0.0718	0.0914
1/7/2011-7/20/2011		AR(1)						R2
Days:131	C	RuTED	EMBI	IFCI	TEDSP	AGG	SPX	
RuTED	(0.0060)*	(0.4252)***	2.2456	(0.3850)	(0.0644)	(0.0279)	0.3623	0.2065
EMBI	0.0003*	(0.0040)	0.0734	(0.0083)	0.0011	(0.0448)	0.0221	0.0353
IFCI	0.0006	(0.0334)	(0.3102)	0.0459	0.0294	(1.7126)**	(0.0576)	0.0786
2/1/2012-8/17/2012		AR(1)						R2
Days:135	C	RuTED	EMBI	IFCI	TEDSP	AGG	SPX	
RuTED	(0.0008)	(0.4085)***	0.7623	(0.5045)	(0.2071)	(1.4231)	0.8348	0.2235
EMBI	0.0004	0.0117*	0.2250*	(0.0018)	(0.0043)	0.5608***	0.0617	0.1489
IFCI	(0.0011)	0.0736*	(0.3572)	(0.0746)	0.0035	2.6409**	0.7570***	0.1018

Note: This table shows the estimation results of VAR in six subsamples segmented by the detected abrupt changes. I only the results of segments with over 100 observations for each time series and equations of the emerging economies.

of quantitative easing, and Central Bank of Russia consecutively reduced the re-financing rate. SPX still had lead effects to both the bond and the stock market in Russia, while AGG had no significant affect.

In the subsample between July 2010 and early 2011, the U.S. Federal Reserve ended its first round of quantitative easing and launched the second round to push up the prices of domestic riskier assets. In this period there was significant positive liquidity spillover effects from the U.S. markets to both the bond and the equity markets in Russia. In other words, the increase of funding liquidity in the U.S. markets led international capital to flow to the Russian markets. The QE2 was not only pushing up the prices of domestic riskier assets but also heating the Russian markets. Meanwhile, both of AGG and SPX only led the Russian bond market.

In the subsample of January 2011 to July 2011, the linkage between the markets in the two economies became weak, when AGG was the only factor that had a lead effect on the Russian markets. Although the Federal Reserve QE2 continued till the end of June 2011, the liquidity spillover was probably offset by the fear of the contagion of the European sovereign debt crisis and the anxiety of U.S. debt crisis. Specifically, AGG had a negative effect on the Russian stock market.

In the last period, from February 2012 to August 2012, the linkage between the markets in the two economies returned to the level before the global crisis, companied by the gradual recovery of the economic situations. SPX had a significant positive forecast power for the Russian stock market; AGG, meanwhile, led significantly both the bond and the stock markets.

Table 3.3: Abrupt Changes Detected between the U.S. -Brazil variables

Abrupt Changes Detected Among the Financial Variable in Brazil					
EMBI	7/24/2007	9/15/2008		1/7/2011	7/29/2011 10/13/2011
IFCI	7/24/2007	9/15/2008	5/28/2010		7/29/2011 12/1/2011
EMBI*ABI	6/7/2007	9/15/2008		1/7/2011	7/29/2011 11/2/2011
Abrupt Changes Detected Among the Financial Variable in U.S.					
TEDSP	7/24/2007	10/21/2008	5/11/2010		7/21/2011 8/26/2011
AGG	6/7/2007	11/26/2008		1/7/2011	11/2/2011
SPX	6/7/2007	7/24/2007	9/15/2008	4/27/2010	8/2/2011 12/1/2011
TEDSP*AGG	6/7/2007	11/5/2008	5/11/2010	1/7/2011	8/25/2011
TEDSP*SPX	7/24/2007	9/15/2008	6/7/2010		8/26/2011
Abrupt Changes Detected Among the Financial Variables Between Brazil and U.S.					
EMBI*TEDSP	7/24/2007	9/15/2008	5/11/2010	1/7/2011	7/21/2011
IFCI*TEDSP	7/24/2007	9/15/2008	5/28/2010		7/21/2011
EMBI*IFCI	7/24/2007	9/15/2008	5/28/2010		7/29/2011 10/28/2011
AGG*SPX	6/7/2007	9/15/2008	4/27/2010		7/29/2011 11/14/2011
IFCI*SPX	7/24/2007	9/15/2008	4/27/2010		7/29/2011 12/1/2011
EMBI*SPX	7/24/2007	9/15/2008			7/29/2011 12/1/2011
IFCI*AGG	6/7/2007	9/15/2008	6/30/2010		7/29/2011 11/2/2011

Note: This table indicates the abrupt changes in the Brazilian markets, in the U.S. markets, and the dependence relationships between the markets in the economies, respectively.

### 3.4.2 U.S. versus Brazil

As mentioned above, in 1999, Central Bank of Brazil substituted the inter-bank basic interest rate and offered rate by the target SELIC rate, which is the overnight lending rate determined by the Central Bank of Brazil's Monetary Policy Committee (COPOM). Therefore, there is no appropriate proxy available to construct a TED spread as the measure of funding liquidity in the Brazilian markets. Nevertheless, I run the VAR without controlling the liquidity in Brazil to see if there is liquidity spillover from the U.S. markets.

The results are reported in a similar pattern as those of the U.S. - Russia linkage. Table 3.3 indicates the abrupt changes in the Brazilian markets, in the U.S. markets, and the dependence relationships between the markets in the economies, respectively. Similar to the abrupt changes detected in the linkage between Russia and U.S., the abrupt change detected in the U.S. bond and stock markets on June 7th 2006 did not affect the Brazilian markets. However, the change in the U.S. stock market on July 24th 2007 indeed had impact on both of the stock and bond markets in Brazil immediately. The influent series of shocking news in the U.S. stock market in mid-September 2008 also triggered abrupt changes in the Brazilian markets. Since then, the structural changes in the Brazilian markets became synchronous as the U.S. markets, such as the group of changes in mid 2010, as an influence of the European sovereign debt crisis, and the group of changes in late July and August 2008, with the global stock market crash. Overall, the abrupt changes detected between financial variables in the Brazil-U.S. linkage had a similar pattern as those detected in the Russia-U.S. linkage, yet more sensitive and synchronous with changes in the U.S. markets.

Table 3.4 presents the estimation results of VAR in six subsamples segmented by some of the detected abrupt changes discussed in Table 3. In the period of July 14th 2006 to June 6th 2007, when the subprime crisis gradually spread from subprime mortgage market to the entire financial sector in the U.S., AGG was the only one factor in the U.S. market negatively leading the Brazilian stock market. During the period when the U.S. financial crisis became the global crisis, i.e., from July 24th 2007 to September 12th 2008, the linkage between Brazil and U.S. significantly increased. As shown in the table, AGG had a one-lag-ahead lead effect to the Brazilian bond market. At the same time, SPX has one-lag-ahead lead effects to both of the bond and the stock markets.



Table 3.4: VAR(P) Coefficients for Brazil-U.S.

7/14/2006-6/6/2007				AR(1)			R2
Days:221	C	EMBI	IFCI	TEDSP	AGG	SPX	
EMBI	0.0006***	0.1503	(0.0063)	(0.0062)	0.1155	(0.0136)	0.0322
IFCI	0.0016	1.5762**	(0.2760)**	0.0235	(2.7869)***	0.3359	0.0720
7/24/2007-9/12/2008				AR(1)			R2
Days:282	C	EMBI	IFCI	TEDSP	AGG	SPX	
EMBI	0.0003	0.0301	0.0049	(0.0012)	0.2410***	0.0522**	0.0630
IFCI	0.0003	0.2951	(0.0780)	(0.0266)	0.2911	0.3691**	0.0451
10/24/2008-4/26/2010				AR(1)			R2
Days:368	C	EMBI	IFCI	TEDSP	AGG	SPX	
EMBI	0.0006**	0.3438***	(0.0238)	(0.0048)	0.1397	0.0547**	0.2428
IFCI	0.0028	0.9336**	(0.3584)**	0.0000	(1.0872)	0.4665***	0.1113
				AR(2)			
		EMBI	IFCI	TED	AGG	SPX	
		0.1624**	(0.0331)**	0.0028	(0.3355)***	(0.0233)	
		0.5712	(0.1159)	0.0225	(0.9173)	0.0466	
6/30/2010-1/6/2011				AR(1)			R2
Days:127	C	EMBI	IFCI	TEDSP	AGG	SPX	
EMBI	0.0004	0.1435	(0.0158)	0.0122*	0.5266***	0.0702**	0.1287
IFCI	0.0024*	0.6351*	0.1534	0.0454**	0.5065	(0.4056)**	0.1390
1/7/2011-7/20/2011				AR(1)			R2
Days:131	C	EMBI	IFCI	TEDSP	AGG	SPX	
EMBI	0.0003	0.2200**	0.0045	0.0022	(0.2907)**	-0.0378	0.0504
IFCI	(0.0003)	0.4327	0.1843	0.0078	(1.2451)*	(0.3644)*	0.0437
12/1/2011-8/17/2012				AR(1)			R2
Days:175	C	EMBI	IFCI	TEDSP	AGG	SPX	
EMBI	0.0003**	0.0974	0.0066	0.0002	0.1321	0.0012	0.0219
IFCI	(0.0002)	(0.1167)	(0.0676)	0.0084	1.0124	0.2863**	0.0159

Note: This table shows the estimation results of VAR in six subsamples segmented by the detected abrupt changes.

In the segment of October 2008 to April 2010, SPX had similar predictive powers to the Brazilian bond and stock markets as the previous segment. Moreover, AGG had a negative predictive power for the Brazilian bond market. In this period, opposite to the monetary expansion in U.S. and Russia, Central Bank of Brazil first increased the basic interest rate to reduce inflationary pressure, and then reduced the rate twice in 2009 to stimulate the economy. However, the basic interest rate was still ranked high in the world and attractive for international capital. As a result, in the period of June 2010 to early 2011, when the Federal Reserve injected liquidity to the U.S. markets by launching QE2, the liquidity spilled over to both markets in Brazil. Simultaneously, AGG and SPX led their corresponding markets, respectively.

In the subsample from January 2011 to July 2011, the U.S. liquidity spillover disappeared, due to the similar reasons mentioned above, i.e., the fear of investors about severity of the European sovereign debt crisis and the anxiety of U.S. debt crisis. Moreover, the negative predictive power of U.S. stock market was at the similar level as that in the previous period. AGG had negative lead effects on both markets in Brazil. In the last segment, i.e., December 2011 to August 2012, SPX was the unique significant factor to the Brazilian stock market, with a positive lead effect.

### **3.4.3 U.S. versus India**

Table 3.5 reports the abrupt changes between the Indian and U.S. markets. Before the global financial crisis breaking out in September 2008, the Indian domestic markets experienced multiple abrupt changes distinct from the U.S. mar-

ket. In particular, the EMBI Global India index (EMBI) changes abruptly on March 29, 2007, when the Indian bond market started to boom. Moreover, the Indian stock market encountered Black Monday on January 22nd 2008, with SENSEX losing over 1480 points in the afternoon trading. Accordingly, the IFCI India index changed abruptly. As a result of the U.S. financial crisis and Indian market changes, there were multiple abrupt changes in the dependence structures between the markets in U.S. and India, which were asynchronous with abrupt changes in the market indexes. These abrupt changes are considered as the result of a change in the global investment climate.

After the beginning of the global financial crisis, the abrupt changes in the Indian markets and in the linkage between the markets in the two economies were almost synchronous as the changes in the U.S. markets. These changes were mainly either triggered by shocking events in the U.S. market or by the European markets.

Table 3.6 indicates the VAR estimation results for the India- U.S. linkage. As we can see in the table, the entire sample is segmented by the detected abrupt changes, and the VAR results are reported for the four segments with over 100 observations. In the period from July 14th 2006 to December 12th 2006, the linkage between the Indian and the U.S. markets were relatively stronger than the linkages between other emerging economies and the U.S. markets at the same period. For example, the U.S. liquidity negatively spilled over to the Indian stock market. Furthermore, SPX had a strong forecast power for both of the bond and stock market in India. AGG also had a positive lead effect on the Indian stock market.

In the period between the Indian stock market crash and the breaking out of

Table 3.5: Abrupt Changes Detected among the financial variables in India-U.S.

Abrupt Changes Detected Among the Financial Variables in India							
InTED	12/13/2006	9/5/2007	9/26/2008	6/14/2010	4/11/2011		2/21/2012
EMBI	3/29/2007		10/10/2008	5/4/2010		9/9/2011	
IFCI		1/21/2008	10/3/2008		3/2/2011		12/22/2011
InTED*EMBI		9/5/2007	9/26/2008	6/23/2010	4/12/2011		4/25/2012
InTED*IFCI	12/13/2006		10/3/2008	7/7/2010	4/6/2011		4/25/2012
EMBI*IFCI		1/21/2008	10/6/2008	6/23/2010		8/30/2011	
Abrupt Changes Detected among the financial variables in U.S.							
TEDSP		7/24/2007	10/21/2008	5/11/2010	7/21/2011	8/26/2011	
AGG	6/7/2007		11/26/2008		1/7/2011		11/2/2011
SPX	6/7/2007	7/24/2007	9/15/2008	4/27/2010		8/2/2011	12/1/2011
TEDSP*AGG	6/7/2007		11/5/2008	5/11/2010	1/7/2011	8/25/2011	
TEDSP*SPX		7/24/2007	9/15/2008	6/7/2010		8/26/2011	
AGG*SPX	6/7/2007		9/15/2008	4/27/2010		7/29/2011	11/14/2011
Abrupt Changes Detected among the financial variables between India and U.S.							
InTED*TEDSP	6/25/2007	9/5/2007	9/15/2008	6/14/2010	4/6/2011	8/18/2011	
InTED*AGG	6/7/2007		9/15/2008	6/18/2010	4/6/2011		4/25/2012
InTED*SPX	6/7/2007	9/5/2007	9/15/2008	4/27/2010	4/6/2011	8/26/2011	
EMBI*TEDSP	5/30/2007	9/21/2007	9/15/2008	6/2/2010	7/21/2011		9/26/2011
EMBI*AGG		9/21/2007	9/15/2008	6/23/2010			12/19/2011
EMBI*SPX	2/27/2007	9/21/2007	9/15/2008	4/27/2010		8/4/2011	12/19/2011
IFCI*TEDSP		7/24/2007	9/15/2008	6/4/2010			9/23/2011
IFCI*AGG			11/14/2008	6/15/2010	3/4/2011		11/2/2011
IFCI*SPX			9/15/2008	4/29/2010		8/4/2011	12/5/2011

Note: This table indicates the abrupt changes in the Indian markets, in the U.S. markets, and the dependence relationships between the markets in the economies, respectively.

global financial crisis, i.e., from January 22nd to September 12th 2008, SPX had the similar predictive power as before, while AGG one-lag-ahead led Indian stock market negatively. In the second subsample, from November 2008 to April 2010, the U.S. Federal Reserve launched the first round of quantitative easing to recover the market functioning. Meantime, India introduced multiple fiscal stimulus packages and monetary expansion policies to keep the financial stability and GDP growth. In this period, although AGG led the Indian bond market,

Table 3.6: VAR(P) Coefficients for India-U.S.

7/14/2006-12/12/2006		AR(1)						R2
Days: 104	C	InTED	EMBI	IFCI	TEDSP	AGG	SPX	
InTED	0.0058	(0.2891)**	1.3885	0.3183	0.1573	(7.0466)*	(0.4781)	0.145
EMBI	0.0003	(0.0049)	(0.1735)	0.0595*	(0.0036)	0.2615	0.1424**	0.1552
IFCI	0.0007	(0.0395)*	(0.0858)	0.1964	(0.0568)*	1.6251**	0.7842***	0.2315
1/22/2008-9/12/2008		AR(1)						R2
Day: 161	C	InTED	EMBI	IFCI	TEDSP	AGG	SPX	
InTED	0.0112	(0.2352)***	1.3590	0.0345	(0.2646)	4.5177	0.6806	0.0916
EMBI	(0.0007)	(0.0052)	(0.0762)	0.0057	0.0097	(0.0230)	0.1474***	0.1679
IFCI	(0.0020)	(0.0127)	0.4797	(0.1998)**	0.0130	(1.1238)*	0.7267***	0.2366
11/26/2008-4/26/2010		AR(1)						R2
Days: 346	C	InTED	EMBI	IFCI	TEDSP	AGG	SPX	
InTED	(0.0090)	(0.0444)	1.2198	(0.6095)	(0.0578)	0.1023	(0.6717)	0.0931
EMBI	0.0006	(0.0050)*	(0.0125)	(0.0221)	(0.0009)	0.3860***	0.0957***	0.086
IFCI	0.0024*	(0.0038)	0.2457	(0.1942)**	(0.0094)	0.5796	0.3528***	0.1001
		AR(2)						
		InTED	EMBI	IFCI	TED	AGG	SPX	
		(0.0836)	4.5341***	(0.7180)	0.1523	5.4616*	(1.0871)**	
		0.0043	(0.1436)*	0.0112	(0.0002)	0.0970	0.0302	
		0.0205*	(0.2631)	(0.0272)	(0.0196)	0.2850	0.3280***	
7/7/2010-1/6/2011		AR(1)						R2
Days: 124	C	InTED	EMBI	IFCI	TEDSP	AGG	SPX	
InTED	0.0065	(0.0736)	1.9534	0.5117	(0.1421)	(5.0014)	(0.0720)	0.0711
EMBI	0.0004	(0.0009)	(0.0941)	0.0233	0.0037	0.4460**	0.1202**	0.0959
IFCI	0.0010	0.0086	0.0801	(0.0113)	0.0332	1.2109**	0.2155	0.1137

Note: This table indicates the VAR estimation results for the India- U.S. linkage. As we can see in the table, the entire sample is segmented by the detected abrupt changes, and the VAR results are reported for the four segments with over 100 observations.

and SPX led the both markets in India, the linkage became weak. In the last period discussed in this table, from July 7th 2010 to January 6th 2011, which was partially overlapping with the U.S. Federal Reserve's second round of quantitative easing, AGG had positive lead effects on both of the markets in India, and SPX also had a positive lead effect on the Indian bond market. Overall, the markets between the two economies linked more closely before the global financial crisis and this linkage became weak after the series of international shocks.

#### **3.4.4 U.S. versus China**

The detected abrupt changes in the U.S.-China linkage are reported in Table 3.7. Before the 2008 global financial crisis, Chinese market had changes triggered by shocking events in the U.S. markets, synchronously with the changes in the U.S. markets. For instance, the announcement of Bear Stearns & Co.'s big losses in its hedge funds was closely related to the abrupt change in the EMBI Global China index (EMBI), which is considered as a benchmark index for the Chinese bond market. On the other hand, the change in the IFCI China index (IFCI), detected on July 27th 2007 was coincident with the U.S. stock market climax. These shocks also led to abrupt changes in the dependence structure between the markets in the two economies. The series of the negative shocking events in mid-September 2008 also triggered abrupt changes in both of the bond and stock markets in China. Moreover, the global stock market crash impacted the both Chinese markets and caused abrupt changes in these markets simultaneously. However, there are frequent abrupt changes detected in the Chinese markets in the period from the end of 2008 to the end of 2009, as well as in the second half 2010, which seemed to be irrelevant to the impact of the European and American

Table 3.7: Abrupt Changes Detected Among the Financial Variables in China-U.S.

Abrupt Changes Detected Among the Financial Variables in China							
ChTED		10/29/2007		11/28/2008	11/24/2009	4/7/2010	1/24/2011
EMBI	6/7/2007		9/19/2008		12/31/2009	11/10/2010	8/5/2011 11/16/2011
IFCI	7/27/2007		9/16/2008	12/18/2008	8/18/2009	6/30/2010	8/5/2011 12/2/2011
ChTED*EMBI		10/29/2007	10/9/2008	1/26/2009	11/24/2009	11/10/2010	8/22/2011
ChTED*IFCI		10/29/2007	10/8/2008	1/26/2009		7/22/2010	
EMBI*IFCI			9/15/2008		2/8/2010	6/30/2010	8/5/2011 12/2/2011
Abrupt Changes Detected Among the Financial Variables in U.S.							
TEDSP	7/24/2007		10/21/2008		5/11/2010	7/21/2011	8/26/2011
AGG	6/7/2007			11/26/2008		1/7/2011	11/2/2011
SPX	6/7/2007 7/24/2007		9/15/2008		4/27/2010	8/2/2011	12/1/2011
TEDSP*AGG	6/7/2007		11/5/2008		5/11/2010	1/7/2011	8/25/2011
TEDSP*SPX	7/24/2007		9/15/2008		6/7/2010	8/26/2011	
AGG*SPX	6/7/2007		9/15/2008		4/27/2010	7/29/2011	11/14/2011
Abrupt Changes Detected Among the Financial Variables Between China and U.S.							
ChTED*TEDSP	8/9/2007	10/29/2007	9/12/2008		11/24/2009	4/7/2010	8/19/2011
ChTED*AGG		10/29/2007	9/29/2008				11/22/2010
ChTED*SPX		10/29/2007	10/9/2008				7/22/2010 12/23/2011
EMBI*TEDSP	7/2/2007		9/15/2008		12/21/2009		7/21/2011 8/24/2011
EMBI*AGG	6/7/2007		9/19/2008		6/14/2010	11/8/2010	11/15/2011
EMBI*SPX	6/7/2007		9/15/2008		5/4/2010	8/1/2011	11/15/2011
IFCI*TEDSP	8/6/2007		9/15/2008		5/26/2010	7/21/2011	10/7/2011
IFCI*AGG			9/16/2008		8/18/2009	6/30/2010	7/29/2011 11/2/2011
IFCI*SPX	7/24/2007		9/15/2008			6/30/2010	8/2/2011 12/2/2011

Note: This table indicates the abrupt changes in the Chinese markets, in the U.S. markets, and the dependence relationships between the markets in the economies, respectively.

sovereign debt crisis.

After detecting the abrupt changes, I run Vector Autoregression for each subsample, and report the estimation results in Table 3.8. Although the abrupt changes in the two economies synchronized before and during the global financial crisis, there is no evidence that there was liquidity spillover from the U.S. markets to the Chinese markets in the sample. Moreover, AGG had no predictive power on either of the Chinese markets until 2011. Specifically, in the subsample segmented by the Bear Stearns' bad news, i.e., from July 14th 2006 to June 6th 2007, SPX had a one-lag-ahead predictive power to the Chinese s-

Table 3.8: VAR(P) Coefficients for China-U.S.

7/14/2006-6/6/2007		AR(1)						R2
Days:221	C	ChTED	EMBI	IFCI	TEDSP	AGG	SPX	
ChTED	0.0035	(0.1025)	(12.4586)	(0.4474)	(0.5150)	20.0392	1.3673	0.0283
EMBI	0.0003**	0.0001	(0.1160)	0.0052	(0.0039)	0.2861	0.0260	0.0316
IFCI	0.0019**	0.0062	(0.1219)	(0.0856)	0.0150	(1.1066)	0.7582***	0.1565
10/29/2007-9/12/2008		AR(1)						R2
Days:216	C	ChTED	EMBI	IFCI	TEDSP	AGG	SPX	
ChTED	0.0006	(0.1110)*	(7.1687)	(0.1026)	(0.2190)	0.1648	(1.9532)	0.0946
EMBI	0.0004	0.0005	(0.1977)	(0.0088)	0.0034	0.0747	0.0025	0.0955
IFCI	(0.0026)	0.0060	0.4986	(0.2133)**	(0.0195)	(0.0946)	0.9453***	0.2856
		AR(2)						
		ChTED	EMBI	IFCI	TED	AGG	SPX	
		(0.0986)*	11.1309	(0.9713)	0.0839	(10.3496)	(0.3690)	
		(0.0007)	(0.2132)	(0.0087)	(0.0061)	0.1522	0.0271	
		(0.0012)	0.2643	0.0139	0.0293	0.1356	0.5178**	
		AR(3)						
		ChTED	EMBI	IFCI	TED	AGG	SPX	
		(0.0213)	3.4296	0.3867	(0.1426)	(0.4951)	1.5566	
		0.0006	(0.1886)	(0.0058)	(0.0029)	0.1548	0.0170	
		0.0069	0.4460	0.0634	0.0131	(0.5366)	0.1627	
1/26/2009-8/17/2009		AR(1)						R2
Days:140	C	ChTED	EMBI	IFCI	TEDSP	AGG	SPX	
ChTED	(0.0211)	(0.1903)*	4.0104	(0.7985)	(0.7930)	(10.9964)	(0.5288)	0.0448
EMBI	0.0002	(0.0002)	(0.0828)	0.0119	(0.0050)	(0.1016)	0.0015	0.0323
IFCI	0.0043*	(0.0027)	(0.0648)	(0.1535)	0.0289	0.1413	0.4819***	0.1672
1/24/2011-7/20/2011		AR(1)						R2
Days:121	C	ChTED	EMBI	IFCI	TEDSP	AGG	SPX	
ChTED	(0.0245)	(0.2446)**	(15.6360)	9.7034	0.5203	9.0114	(3.7801)	0.0964
EMBI	0.0002	0.0002	(0.0349)	0.0162	0.0002	0.7541***	(0.0083)	0.4541
IFCI	(0.0004)	0.0023	(0.0341)	(0.0123)	0.0285	(0.0729)	0.6039***	0.2323
12/23/2011-8/17/2012		AR(1)						R2
Days:159	C	ChTED	EMBI	IFCI	TEDSP	AGG	SPX	
ChTED	0.0452	(0.2238)***	(42.6603)	14.1181**	(2.5066)	3.6825	(16.4704)*	0.1047
EMBI	0.0003*	(0.0002)	0.0786	(0.0374)**	0.0014	0.2059*	0.0870***	0.1178
IFCI	(0.0006)	(0.0007)	0.0271	(0.1749)*	(0.0120)	1.9828***	0.8028***	0.2122

Note: This table indicates the VAR estimation results for the China- U.S. linkage. As we can see in the table, the entire sample is segmented by the detected abrupt changes, and the VAR results are reported for the four segments with over 100 observations.



tock market. During the period from October 29th 2007 to September 12th 2008, SPX led the Chinese stock market with both one-lag and two-lag ahead predictive powers. In the third subsample, which is from January 26th 2009 to August 17th 2009, SPX had a weaker predictive power on the Chinese stock market than that before the global financial crisis, as the U.S. Federal Reserve implemented the first round of quantitative easing.

During the segment between January 24th 2011 and July 20th 2011, partially overlapping with the second round of U.S. quantitative easing, AGG and SPX had one-lag-ahead lead effect on the Chinese bond and Stock markets, respectively. These enhanced predictive powers were probably due to the recovery of the U.S. credit market and the equity market, stimulated by the two rounds of quantitative easing. In the last segment between December 23rd 2011 and August 17th 2012, the predictive power of both AGG and SPX continued to enhance, with lead effects on both the bond and stock markets in China.

### **3.4.5 U.S. versus South Africa**

Table 3.9 shows the detected abrupt changes in the linkage between U.S. and South Africa. The first abrupt change in the EMBI Global South Africa index (EMBI) was coincident with the first changing point in AGG and SPX, possibly triggered by the shocking news of big losses in Bear Stearns' hedge funds. Before the global financial crisis spillover from the U.S. markets, the TED spread in South Africa changed abruptly in May 2008, leading its dependence relationships with other financial variables of South Africa and U.S. change abruptly. Then the shocking news in mid-September impacted the IFCI South Africa

Table 3.9: Abrupt Changes Detected among the Financial Variables in South Africa-U.S.

Abrupt Changes Detected Among the Financial Variables in South Africa						
SATED		5/28/2008		3/23/2009	12/6/2010	12/7/2011
EMBI	6/7/2007		10/9/2008		6/15/2010	7/29/2011
IFCI			9/19/2008		4/28/2010	8/4/2011
SATED*EMBI	5/18/2007	5/22/2008	10/9/2008	3/23/2009	12/9/2010	1/31/2012
SATED*IFCI		5/29/2008	10/6/2008	3/23/2009	12/6/2010	11/23/2011
EMBI*IFCI			10/6/2008		4/22/2010	7/29/2011
Abrupt Changes Detected Among the Financial Variables in U.S.						
TEDSP		7/24/2007	10/21/2008	5/11/2010	7/21/2011	8/26/2011
AGG	6/7/2007		11/26/2008		1/7/2011	11/2/2011
SPX	6/7/2007	7/24/2007	9/15/2008	4/27/2010	8/2/2011	12/1/2011
TEDSP*AGG	6/7/2007		11/5/2008	5/11/2010	1/7/2011	8/25/2011
TEDSP*SPX		7/24/2007	9/15/2008	6/7/2010		8/26/2011
AGG*SPX	6/7/2007		9/15/2008	4/27/2010	7/29/2011	11/14/2011
Abrupt Changes Detected Among the Financial Variables Between South Africa and U.S.						
SATED*TEDSP	5/18/2007	7/27/2007	5/29/2008		12/21/2010	
SATED*AGG	10/10/2006	11/28/2007	5/22/2008	6/15/2009	1/18/2011	12/7/2011
SATED*SPX	10/10/2006	11/28/2007	5/29/2008	3/25/2009	12/6/2010	12/12/2011
EMBI*TEDSP	6/7/2007		9/12/2008	5/21/2010	7/21/2011	10/7/2011
EMBI*AGG	6/7/2007		9/29/2008	6/15/2010	7/29/2011	11/4/2011
EMBI*SPX	6/7/2007		10/7/2008	4/27/2010	7/29/2011	12/1/2011
IFCI*TEDSP		7/25/2007	9/12/2008	5/27/2010	7/21/2011	9/28/2011
IFCI*AGG			9/19/2008	6/16/2010	7/29/2011	11/4/2011
IFCI*SPX		7/24/2007	9/18/2008	4/27/2010	8/4/2011	12/1/2011

Note: This table indicates the abrupt changes in the South African markets, in the U.S. markets, and the dependence relationships between the markets in the economies, respectively.

index (IFCI) immediately and also dragged down the South African bond index within a month. Similar to Russia and Brazil, the bond and stock markets in South Africa also experienced abrupt changes between April and July 2010, probably as the result of the European sovereign debt crisis, and abrupt changes in late July and early August 2011, triggered by the anxiety of the deterioration of the European sovereign debt crisis and the possibility of the U.S. debt crisis. In addition, the time series of TED spread had abrupt changes on March 23rd 2009 and December 6th 2010, with abrupt changes in its dependence relations with other financial variable of U.S. and South Africa.

Table 3.10 shows the estimation results of VAR for financial variables of U.S. and South Africa. In the period of October 10th 2006 to May 17th 2007, AGG and SPX Granger-caused the subsequent changes in the South African bond market and stock market, respectively. Then in the following period of November 28, 2007 to May 22nd 2008, AGG and SPX both had predictive powers on the two markets in South Africa. Similar to the Russia-U.S. dependence structure changing during the first round of U.S. quantitative easing, in the subsample between June 15th 2009 and April 26th 2010, the linkage between South Africa and U.S. became weaker than before, with merely the lead effect of SPX to the South African markets.

In the period from mid-June 2010 to early December 2010, U.S. liquidity had a positive lead effect on the South African stock market, which was very likely to be the result of U.S. QE 2. That is, the U.S. quantitative easing pushed up the purchase of riskier assets both in U.S. and in emerging markets. Meanwhile, the predictive powers of AGG and SPX on South African markets increased, which may also be related to the stimulus of quantitative easing. In the last segment from January 2012 to August 2012, the linkage between U.S. and South Africa returns to the level before the global financial crisis, with SPX leading the stock market and AGG leading the both markets in South Africa.

In sum, most of abrupt changes in the linkage of U.S. and an emerging economy were triggered by specific negative shocking events or as cumulative result of series of negative shocking events. The financial variables for stock and bonds markets were more sensitive to the shocks than the variables for liquidity. The empirical result further shows that financial crisis started from the U.S. domestic market in 2007 and became global in September 2008, when abrupt changes

Table 3.10: VAR(P) Coefficients for South Africa-U.S.

10/10/2006-5/17/2007		AR(1)						R2
Days:149	C	SATED	EMBI	IFCI	TEDSP	AGG	SPX	
SATED	0.0008	(0.0505)	1.4323	(0.4968)**	0.0231	(0.9217)	1.3013**	0.0455
EMBI	0.0004***	(0.0016)	(0.1146)	0.0121	(0.0046)	0.4504***	0.0076	0.1504
IFCI	0.0018	(0.0420)*	(1.6079)**	(0.0700)	0.0035	(0.0997)	0.8316***	0.1545
11/28/2007-5/22/2008		AR(1)						R2
Days:119	C	SATED	EMBI	IFCI	TEDSP	AGG	SPX	
SATED	0.0078**	0.0007	1.2331	0.1477	(0.0053)	(1.0152)	(0.3078)	0.0519
EMBI	0.0000	0.0088	(0.0637)	0.0077	0.0026	0.1593*	0.0859***	0.1744
IFCI	0.0003	(0.0418)	(0.7171)	(0.1464)	(0.0315)	1.1139*	0.8977***	0.3110
6/15/2009-4/26/2010		AR(1)						R2
Days:213	C	SATED	EMBI	IFCI	TEDSP	AGG	SPX	
SATED	(0.0003)	(0.0464)	0.3783	0.0569	0.0164	(0.2793)	(0.0903)	0.0128
EMBI	0.0005**	(0.0060)	0.0877	0.0198	(0.0004)	0.0856	0.0477*	0.0712
IFCI	0.0010	(0.0673)	(0.1135)	(0.0748)	(0.0082)	0.3712	0.5254***	0.0847
6/15/2010-12/3/2010		AR(1)						R2
Days:136	C	SATED	EMBI	IFCI	TEDSP	AGG	SPX	
SATED	(0.0004)	(0.0468)	(1.4094)	0.5024**	0.0399	1.6733	(0.2926)	0.0860
EMBI	0.0004	0.0117	0.2796**	(0.0224)	0.0048	0.5266***	0.0702**	0.2791
IFCI	0.0027*	(0.0513)	(0.1804)	(0.0375)	0.1007***	0.8661	0.3300**	0.2024
1/31/2012-8/17/2012		AR(1)						R2
Days:135	C	SATED	EMBI	IFCI	TEDSP	AGG	SPX	
SATED	(0.0007)	(0.0118)	0.0369	(0.0546)	0.0150	0.0349	0.0370	0.0150
EMBI	0.0004	0.0129	0.3161***	0.0154	(0.0033)	0.5033**	(0.0205)	0.1708
IFCI	(0.0003)	(0.0763)	0.0516	(0.1894)	0.0094	2.2665**	0.6569**	0.0776

Note: This table indicates the VAR estimation results for the South Africa- U.S. linkage. The entire sample is segmented by the detected abrupt changes, and the VAR results are reported for the four segments with over 100 observations.

were worldwide synchronous. The U.S. liquidity spilled over to Russia, Brazil, and South Africa temporarily in 2010 and early 2011, as the U.S. Federal Reserve implemented the first two rounds of quantitative easing. Then the spillover effect disappeared, probably due to the fear of investors about the contagion of the European sovereign debt crisis and the anxiety of U.S. debt crisis. Compared to the spillover effect of U.S. liquidity, the lead effects of the U.S. stock and bond markets were more significant. For most of emerging economies, the linkages with U.S. reduced in some period after the global financial crisis, and returned to previous levels soon afterwards, which could be the result of policy responses of different economies. On the one hand, the U.S. economic news affected the global markets and led the synchronous structural changes; on the other hand, the lead effects of the U.S. bond and equity markets were much more significant than the spillover effect of liquidity. Therefore, the financial spillovers from U.S. to emerging markets were more likely through the correlated information channel than the liquidity channel.

Moreover, the two-step method can capture dynamic patterns of the linkage with the two types of structural changes. In the first step, the abrupt changes in the covariance structures are detected and the entire sample is segmented by the detected changing points. Then time-varying VAR approximates the smoothly changed coefficients by polynomials. In this model, the smooth changing multivariate process is assumed to be locally stationary, i.e., underlying assumption of the model does not allow any abrupt change in the multivariate process. The estimated results show there are multiple abrupt changes between the linkage between the U.S. and the emerging markets, and the coefficients of lag variables are constant within each subperiods. Instead, if the abrupt changes are ignored and the time-varying VAR is applied directly, then the assumption is violated

and the statistical inferences are unreliable. To the best of my knowledge, there is no existing method can capture the two types of the covariance structural changes simultaneously.

### **3.5 Conclusion**

In this paper, I empirically investigate the linkages between the U.S. markets and the emerging markets to identify the global liquidity and financial spillover after the 2009 global financial crisis. A two-step method is adopted to capture the dynamic patterns and structural changes in the linkages between the bond and stock markets in U.S. and BRICS. The TED spreads are constructed to control the funding liquidity in the markets in each economy. The U.S. Dollar-denominated bond and stock indexes are adopted to be the benchmarks of the emerging markets. The results show that most abrupt changes in both the U.S. and BRICS markets were due to specific shocking events in the U.S. markets, and the abrupt changes were globally synchronous after the global financial crisis. Furthermore, there were temporary liquidity spillover effects from U.S. to some of the emerging markets, as U.S. Federal Reserve implemented the second round of quantitative easing. The spillover effects were probably offset by the fear of investors about the contagion of the European sovereign debt crisis and the anxiety of U.S. debt crisis. Overall, the lead effects of the U.S. bond and equity markets were much more significant than the spillover effects of liquidity. Thus I also conclude that the financial spillovers were more likely through the correlated-information channel than the liquidity channel.

CHAPTER 4

GENERALIZED SPECTRAL ESTIMATION OF TIME SERIES  
CONDITIONAL MOMENT RESTRICTIONS MODELS WITH INFINITE  
DIMENSIONAL CONDITIONING SET

## 4.1 The Generalized Spectral Estimator

The framework we are considering in this study is a time series model defined by the following conditional moment restriction:

$$E[\rho(Z_t; \theta_0) \mid \mathcal{F}_{t-1}] = 0 \quad (4.1)$$

where  $Z_t$  is a time series vector,  $\theta_0 \in \Theta \subset \mathbb{R}^p$  the unique  $p \times 1$  vector of parameters to make equation (4.1) hold,  $\rho(Z_t; \theta)$  a  $J \times 1$  vector of functions, and  $\mathcal{F}_{t-1} = \{X_{t-1}, X_{t-2}, \dots\}$  with  $X_{t-j}$  as a  $q \times 1$  random vector the  $\sigma$ -algebra defined by the time  $t$  information set. Equation (4.1) defines the parameter value of interest  $\theta_0$ , which is unknown to econometricians and also what we will estimate. The function  $\rho(\cdot; \cdot)$  is supposed to be known. Moreover, the information set  $\mathcal{F}_{t-1}$  contains all the exogenous variables and even the functions of lagged variables,  $\rho(Z_{t-j}; \theta_0)$  ( $j = 1, 2, \dots$ ) for example. Such a framework is general enough to cover most models which can be represented by conditional moment restrictions as special cases, including IV models with  $\mathcal{F}_{t-1}$  as the  $\sigma$ -algebra generated by the exogenous variables, nonlinear dynamic regression models with  $\rho(Z_t; \theta_0)$  understood as the errors, and rational expectations models such as consumption based asset pricing models defined by the Euler equation.

To estimate the model (4.1) consistently, there are two issues we have to take into account. The first, also our focus in this study, is related to the time series

information set  $\mathcal{F}_{t-1}$ . As can be seen from (4.1), variables at all the lag orders are included in the conditional information set. The implication of this for the estimation method is that the lag order of variables should increase with the sample size of observations to make full use of conditional information in  $\mathcal{F}_{t-1}$ . This has to be the case for models with the conditional information set  $\mathcal{F}_{t-1}$  containing variables of all lag orders. The rational expectations models, such as consumption based asset pricing models for which (4.1) is the Euler equation, are a standard example (Hansen and Singleton, 1982; Singleton, 2006, Section 2.3.2).<sup>1</sup>

Second, the model is formulated in terms of conditional instead of unconditional moments. For example, suppose the information set  $\mathcal{F}_{t-1}$  contains only the one-period-lagged variable  $X_{t-1}$ . Then the model (4.1) is now

$$E[\rho(Z_t; \theta_0) | X_{t-1}] = 0 \quad (4.2)$$

via the conditional mean of  $\rho(Z_t; \theta_0)$  on  $X_{t-1}$ . This further implies the following unconditional moment condition

$$E[\rho(Z_t; \theta_0) X_{t-1}] = 0 \quad (4.3)$$

which underlies the generalized method of moments (GMM) as the commonly employed method to estimate the model (4.2). However, it is well known that the conditional moment restriction implies an infinite number of unconditional ones. In this simple example, (4.2) actually implies that  $E[\rho(Z_t; \theta_0) g(X_{t-1})] = 0$  for any function  $g(\cdot)$ . There is a big gap between the model (4.2) and (4.3) especially for nonlinear  $\rho(\cdot; \cdot)$ . Consequently, the uniquely identified parameter

---

<sup>1</sup>Another one is the diffusion model which, according to Song (2009), is identified by a MD property through an infinitesimal operator based approach. The MD property in turn delivers the identification of the diffusion model in terms of (4.1) with the conditioning variables  $X_{t-j}$  as the lagged variables of all orders from the original process.



$\theta_0$  in (4.2) may not be identified by the unconditional moment restriction (4.3). That is, (4.3) may hold for several parameter values although the conditional moment restriction (4.2) just holds for a single value. See Domínguez and Lobato(2004) for two simple but illuminating examples of models only identified by the conditional but not unconditional moment restrictions. Therefore, estimators based on the unconditional moment restrictions, which are not equivalent to conditional ones, may not be consistent at all.

While there are many different estimators for special cases of model (4.1), no estimator has been proposed, to the best of our knowledge, for the general model (4.1) incorporating both the infinite dimensional conditioning set and conditional moment restrictions. For example, models defined by conditional moment restrictions for cross section data with  $\mathcal{F}_{t-1}$  containing only exogenous variables or those for time series data with  $\mathcal{F}_{t-1}$  containing only variables of finite fixed lag orders are analyzed repeatedly in econometrics literature. Obviously, the first issue disappears for this case since the infinite dimensional conditioning set degenerate to a finite dimensional one. Many estimators have been proposed for such a model but most of them do not address the conditional identification problem associated with the conditional moment restrictions; see Amemiya (1974, 1977), Hansen(1982), Newey (1990, 1993), and Robinson (1987, 1991). To tackle the conditional identification problem, several different methods are suggested including Domínguez and Lobato(2004) via the indicator function, Donald, Imbens and Newey(2003) based on a sequence of approximating functions such as splines or power or Fourier series<sup>2</sup>, and Kitamura,

---

<sup>2</sup>The idea of Donald, Imbens and Newey(2003) that the number of unconditional moment conditions increases with the sample size at a slower growth rate can be adapted for the case of infinite conditioning set we consider in this study. This is actually the approach we employ in a former version. But such an estimator still only contains parts of the conditional information and is inconvenient to implement in practice because a high dimensional integration is involved.

Tripathi and Ahn(2004) through localized empirical likelihood. All these estimators, however, will not be consistent for the general model (4.1) because all the lagged variables from the information set  $\mathcal{F}_{t-1}$  should be included for the identification but only a limited number of them are taken into account. The following example illustrates the problem:

**Example 1** Suppose  $(Y_t, X_t)'$  follows a dynamic regression model defined by

$$Y_t = \alpha_{1_0}X_t + \alpha_{2_0}X_{t-1} + \epsilon_t$$

where  $E[\epsilon_t|\mathcal{F}_t] = 0$ ,  $X_{t-1} = \beta_0 X_t + e_t$ , with  $e_t \sim N(0, 1)$  and  $\beta_0$  a *known* parameter, and  $\mathcal{F}_t$  is the information set generated by  $\{X_t, X_{t-1}, \dots\}$ . Equivalently this model can also be identified by  $E[Y_t|\mathcal{F}_t] = E[Y_t|X_t, X_{t-1}] = \alpha_{1_0}X_t + \alpha_{2_0}X_{t-1}$ . Suppose now a mis-specified conditional information set,  $\mathcal{F}'_t = \{X_t\}$ , is used as the basis for estimating  $(\alpha_1, \alpha_2)$ , then the mis-specified identification condition will be:

$$\begin{aligned} 0 &= E[\epsilon_t|\mathcal{F}'_t] \\ &= E[\epsilon_t|X_t] \\ &= E[E[Y_t|\mathcal{F}_t] - \alpha_1 X_t - \alpha_2 X_{t-1}|X_t] \\ &= (\alpha_{1_0} - \alpha_1)X_t + (\alpha_{2_0} - \alpha_2)E[X_{t-1}|X_t] \\ &= [(\alpha_{1_0} - \alpha_1) + \beta_0(\alpha_{2_0} - \alpha_2)]X_t \end{aligned}$$

where the third equality follows from the law of iterated expectation. This implies that  $\alpha_1 + \alpha_2\beta_0 = \alpha_{1_0} + \alpha_{2_0}\beta_0$  which is a linear equation with two unknowns  $(\alpha_1, \alpha_2)$ . In other words, any vector  $(\alpha_1, \alpha_2)$  satisfying this equation will be a solution of the parameter values and there are actually an infinite number of them consisting of points on a line. Hence, the moment condition based on the mis-specified conditional information, which does

not make full use of the conditioning set, cannot identify the true parameters  $(\alpha_{1_0}, \alpha_{2_0})$ . Consequently, estimators based on this moment condition are not consistent at all.

The simple Example 1 shows that neglecting one conditioning variable from the full conditional information set will result in no identification of the parameters and thus inconsistency of the estimators. Similarly, when the conditional information set is infinite dimensional and we have no prior information about which conditioning variables are needed for the identification, it is better to employ an estimator utilizing the infinite dimensional conditioning set completely. Berkowitz(2001) does propose an estimator making full use of the infinite dimensionality of the information set  $\mathcal{F}_{t-1}$  by the power spectrum in the frequency domain. The idea is to replace the conditional moment condition  $E[\rho(Z_t, \theta_0)|\mathcal{F}_t] = 0$  by the unconditional moment restriction

$$E\left[\rho(Z_t, \theta_0) \times \rho(Z_{t-j}, \theta_0)\right] = 0 \text{ for } j = 1, 2, \dots \quad (4.4)$$

which further imply, in terms of frequency domain concepts, that the spectral density  $g(\omega; \theta) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} \gamma(j; \theta) e^{-ij\omega}$ , with  $\gamma(j; \theta) = \text{Cov}[\rho(Z_t, \theta), \rho(Z_{t-j}, \theta)]$  as the autocovariance function at lag  $j$ , will be a flat spectrum equal to  $g_0(\omega; \theta) = \gamma(0; \theta) / (2\pi)$ . Then a minimum distance type estimator is proposed by minimizing the distance between the estimated general spectral density and flat spectral density under the identification condition, i.e., the Cramér-Von Mises (CVM) test statistic in Durlauf (1991).

However, the characterization through power spectrum which is the Fourier transform of the auto-covariance function, is only equivalent to the unconditional moment conditions as in (4.3) and subject to the conditional identification

problem caused by the nonlinear dependence. The following example illustrates the problem:

**Example 2** Consider the following AR(1) model:

$$X_t = \theta_0 X_{t-1} + \epsilon_t \text{ with } E[\epsilon_t | \mathcal{F}_{t-1}] = 0$$

where  $\mathcal{F}_{t-1} = \{X_{t-1}, X_{t-2}, \dots\}$ . Thus, in terms of model (4.1) terminology,  $\rho(Z_t, \theta_0) = \epsilon_t(\theta_0) = X_t - \theta_0 X_{t-1}$  and the moment conditions employed by Berkowitz(2001) are:

$$E[\epsilon_t(\theta) \epsilon_{t-j}(\theta)] = 0, \text{ for } j = 1, 2, \dots$$

i.e.

$$\begin{aligned} E[\epsilon_t(\theta) \epsilon_{t-j}(\theta)] &= E[(X_t - \theta X_{t-1})(X_{t-j} - \theta X_{t-j-1})] \\ &= E[E[(X_t | \mathcal{F}_{t-1})] - \theta X_{t-1})(X_{t-j} - \theta X_{t-j-1})] \\ &= (\theta - \theta_0)^2 E[X_{t-1} X_{t-j-1}] + (\theta_0 - \theta) E[X_{t-1} \epsilon_{t-j}] \end{aligned}$$

which implies that for all  $j > 0$ , either  $\theta = \theta_0$  or  $\theta = \theta_0 + \frac{E[X_{t-1} \epsilon_{t-j}]}{E[X_{t-1} X_{t-j-1}]}$ . As long as the process  $\{X_t\}$  is such that  $\frac{E[X_{t-1} \epsilon_{t-j}]}{E[X_{t-1} X_{t-j-1}]} \neq 0$ , two values are available for the parameters and only one of them is the true value. In this sense, Berkowitz's(2001) estimator based on the linear dependence measure  $\gamma(j; \theta)$  cannot identify the parameters.

In the following, we will propose an estimator for the general model (4.1) free of both identification problems due to the conditional moment restrictions and infinite dimensionality of the conditioning information set. For simplicity, we consider the case with  $J = 1$  and  $q = 1$ . The moment conditions underlying our estimator are:

$$E[\rho(Z_t, \theta_0) | X_{t-j}] = 0 \text{ for } j = 1, 2, \dots \quad (4.5)$$

implied by (4.1). It is easily seen that (4.5) captures the pairwise relationship between  $\rho(Z_t, \theta_0)$  and lagged variables of all orders although it is not completely equivalent to (4.1). In order to be closer to (4.1), one may alternatively consider

$$E [\rho(Z_t, \theta_0) | X_{t-1}, X_{t-2}, \dots, X_{t-P}] = 0$$

with  $P$  tending to infinity with the sample size. But such a approach brings up some difficulties especially because a computationally obstacle,  $P$ -dimensional integration, is involved. The approach employing (4.5) avoids the high dimensional integration while being able to take into account all lags available in the sample data. It represents a nice compromise between generality and simplicity. This idea has already been utilized in studies on testing problems initialized by Hong(1999) and followed by Hong and Lee(2005), Escanciano(2006), Escanciano and Velasco(2006), and so on. Here we employ it in an estimation framework. Just a simple comparison can reveal that the moment conditions (4.5) we employ are more general than (4.4) in Berkowitz(2001) in two respects: first, the instruments in (4.4) are only special cases of those in (4.5); second, (4.5) is in terms of conditional moments while (4.4) unconditional moments which are subject to the identification problem caused by linear dependence measure.

A conditional moment dependence measure can be used to characterize the conditional moment conditions (4.5):

$$\gamma_j(u; \theta) = E [\rho(Z_t, \theta) \times e^{iuX_{t-j}}] \quad (4.6)$$

It can be viewed as a generalization of the standard auto-covariance to capture the conditional mean dependence in a nonlinear time series framework. The "generalization" here has two-fold meanings: for the first place, it can characterize the nonlinear dependence between  $\rho(Z_t, \theta)$  and  $X_{t-j}$  as the generalization of  $E [\rho(Z_t, \theta) \times X_{t-j}]$  which is only for linear dependence; for the second,  $e^{iuX_{t-j}}$

instead of  $\rho(Z_{t-j}, \theta_0)$  generalizes the "auto" in (4.5). From Bierens(1982),

$$\gamma_j(u; \theta) = 0 \text{ for all } j \neq 0 \text{ and } u \in \mathbb{R} \text{ almost everywhere(a.e.)} \Leftrightarrow (4.5) \quad (4.7)$$

The sample counterpart of  $\gamma_j(u; \theta)$  based on a sample  $\{Z_t, X_t\}_{t=1}^n$  is:

$$\widehat{\gamma}_j(u; \theta) = \frac{1}{n-j} \sum_{t=j+1}^n \left( \rho(Z_t, \theta) - \bar{\rho}_{n-j} \right) e^{iuX_{t-j}},$$

with

$$\bar{\rho}_{n-j} = \frac{1}{n-j} \sum_{t=j+1}^n \rho(Z_t, \theta) \quad (4.8)$$

Define  $\gamma_{-j}(\cdot; \theta) = \gamma_j(\cdot; \theta)$  and then consider the Fourier transform of  $\gamma_j(u; \theta)$ :

$$f(\omega, u; \theta) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j(u; \theta) e^{-ij\omega}, \text{ for } \omega \in [-\pi, \pi] \text{ and } u \in \mathbb{R} \quad (4.9)$$

$f(\omega, u; \theta)$  is similar to the generalized spectral derivative in Hong(1999) and Hong and Lee(2005) with the difference that the lagged variables  $X_{t-j}$  are replaced by  $\rho(Z_{t-j}, \theta)$  for their studies. It exists if

$$\sup_{u \in \mathbb{R}} \sum_{j=-\infty}^{\infty} |\gamma_j(u; \theta)| < \infty,$$

which holds under proper mixing conditions(see Hong(1999) for details).

By (4.5) and (4.7), the identification of the model is equivalent to

$$f_0(\omega, u; \theta_0) = \frac{1}{2\pi} \gamma_0(u; \theta_0) \quad (4.10)$$

That is, (4.10) is a model-implied restriction on  $f(\omega, u; \theta)$ . Therefore, a class of estimators can be obtained by minimizing the distance between the estimator of general  $f(\omega, u; \theta)$  and that of the model implied  $f_0(\omega, u; \theta)$  via some measure. Hong and Lee's(2005) kernel based estimator can be used here but then the estimator for  $\theta_0$  will also depend on the kernels chosen and bandwidth choices.

Hence, we turn to an alternative method via the generalized spectral distribution function analogous to Durlauf's(1991) idea for testing. The generalized spectral distribution function is defined as

$$H(\lambda, u; \theta) = 2 \int_0^{\lambda\pi} f(\omega, u; \theta) d\omega \text{ for } \lambda \in [0, 1] \text{ and } u \in \mathbb{R}$$

That is,

$$H(\lambda, u; \theta) = \gamma_0(u; \theta) \lambda + 2 \sum_{j=1}^{\infty} \gamma_j(u; \theta) \frac{\sin j\pi\lambda}{j\pi} \text{ for } \lambda \in [0, 1] \text{ and } u \in \mathbb{R}$$

which can then be estimated by the sample analog:

$$\widehat{H}(\lambda, u; \theta) = \widehat{\gamma}_0(u; \theta) \lambda + 2 \sum_{j=1}^{n-1} \left(1 - \frac{j}{n}\right)^{1/2} \widehat{\gamma}_j(u; \theta) \frac{\sin j\pi\lambda}{j\pi} \quad (4.11)$$

where  $(1 - j/n)^{1/2}$  is a finite sample correction factor as in Hong(1999). Similarly, the model implied generalized spectral distribution function is

$$H_0(\lambda, u; \theta) = \gamma_0(u; \theta) \lambda$$

which can be estimated by

$$\widehat{H}_0(\lambda, u; \theta) = \widehat{\gamma}_0(u; \theta) \lambda \quad (4.12)$$

Now, the generalized spectral estimator (GSE in this paper) we will propose is defined as

$$\widehat{\theta} = \arg \min_{\theta} D_n^2(\theta),$$

where  $D_n^2$  is the actually the sample analog of

$$\begin{aligned} D^2(\theta) &= \int_{\mathbb{R}} \int_0^1 |H(\lambda, u; \theta) - H_0(\lambda, u; \theta)|^2 W(du) d\lambda \\ &= \sum_{j=1}^{\infty} \frac{1}{(j\pi)^2} \int_{\mathbb{R}} |\gamma_j(u; \theta)|^2 W(du) \end{aligned} \quad (4.13)$$

where  $W(\cdot)$  is a weighting function satisfying some conditions (see Section 3 for them). Considering the process  $S_n(\lambda, u; \theta) = (\frac{n}{2})^{\frac{1}{2}} \{\widehat{H}(\lambda, u; \theta) - \widehat{H}_0(\lambda, u; \theta)\} = \sum_{j=1}^{n-1} (n-j)^{\frac{1}{2}} \widehat{\gamma}_j(u; \theta) \frac{\sqrt{2} \sin j\pi\lambda}{j\pi}$  as the sample analog of  $H(\lambda, u; \theta) - H_0(\lambda, u; \theta)$ ,

$$\begin{aligned} D_n^2(\theta) &= \int_{\mathbb{R}} \int_0^1 |S_n(\lambda, u; \theta)|^2 W(du) d\lambda \\ &= \sum_{j=1}^{n-1} (n-j) \frac{1}{(j\pi)^2} \int_{\mathbb{R}} |\widehat{\gamma}_j(u; \theta)|^2 W(du) \end{aligned} \quad (4.14)$$

This estimator actually minimizes the Cramér-Von Mises (CVM) test statistic described in Escanciano and Velasco (2006). Unlike minimizing the  $L^2$ -metric in Hong (1999) and Hong and Lee (2005), our estimator does not involve choosing a kernel function and any user-chosen number and hence is computationally simple.

The “moment conditions” associated with our proposed estimator are  $E \int_0^{\lambda\pi} [f(\omega, u; \theta) - f_0(\omega, u; \theta)] d\omega = 0$  for all  $\lambda \in (0, 1)$  and  $u \in \mathbb{R}$ . Interestingly, these are equivalent to

$$E \frac{1}{\pi} \sum_{j=1}^{\infty} \gamma_j(u; \theta) \frac{\sin j\pi\lambda}{j} \text{ for all } \lambda \in (0, 1) \text{ and } u \in \mathbb{R}$$

This equivalence makes clear that the generalized spectral estimator in (4.14) is actually obtained by setting the conditional moment dependence measure in (4.6) as close as possible to zero. Not only are all the lag orders of the conditioning variables included in this estimator, but also the nonlinear dependence is captured free of the identification problem.

## 4.2 Asymptotic theory

This section provides sufficient conditions for the consistency of the generalized spectral estimator in (4.13) when the model (4.1) is correctly specified. The



following regularity conditions on the data-generating process are imposed:

**Assumption 4.1** The unknown parameter vector  $\beta_0$  belongs the parameter space  $\Theta$  which is a compact subset of  $\mathbb{R}^p$ .

**Assumption 4.2**  $\left\{ \begin{pmatrix} \rho(Z_t, \theta) \\ X_t \end{pmatrix} : t = 1, 2, \dots \right\}$  is a sequence of random vectors defined on a probability space and  $\rho(Z_t, \theta)$  is continuous in  $\beta$ .

The function  $\rho(\cdot, \cdot)$  is usually given by the first-order conditions of the econometric model or by the specification of the agent's preference, budget restrictions and so on in the rational expectations framework.

**Assumption 4.3**  $\begin{pmatrix} \rho(Z_t, \theta) \\ X_t \end{pmatrix}$  is strong mixing of size  $-\frac{r}{r-1}$  for some  $r > 1$  and possesses finite second order moments, for all  $t$  and all  $\beta \in \Theta$ .

**Assumption 4.4**  $f(\omega, u; \theta)$  in (4.9) exists and is continuous on  $\Theta \times \mathbb{R}$ .

A sufficient condition for this assumption has been discussed in Section 2. Also see Priestley(1981) for more reference.

**Assumption 4.5** There exists a unique  $\theta_0$  such that the generalized spectral density function  $f(\omega, u; \theta)$  is a constant equal to  $f_0(\omega, u; \theta)$  for each fixed  $u$ .

This is the identification condition underlying the generalized spectral estimator in (4.13). It is analogous to Hansen's (1982) requirement that the population moments implied by the model have a unique zero for GMM.

**Assumption 4.6** Both  $E [\rho(Z_t, \theta)]^4$  and  $E (X_t)^4$  are finite for all  $t$  and all  $\theta \in \Theta$ .

Assumption 4.6 is a standard condition for establishing asymptotic properties of the sample generalized spectral distribution  $\widehat{H}(\lambda, u; \theta)$  in (4.11).

Another assumption we need is related to the Cramér-Von Mises criterion we employ to propose the generalized spectral estimator in (4.13).

**Assumption 4.7**  $E \left| \widehat{H}(\lambda, u; \theta) - \widehat{H}_0(\lambda, u; \theta) \right|^8$  is finite for  $\theta \in \Theta$ ,  $\lambda \in (0, 1)$  and  $n \geq 1$ .

**Assumption 4.8**  $W(\cdot)$  is a probability measure on  $\mathbb{R}$ , absolutely continuous with respect to Lebesgue measure.

Any probability measure can be chosen for the weighting function, among which normal or exponential distribution functions may be preferred because a closed-form  $D_n^2$  may be easily obtained which makes the implementation of the estimator computationally convenient in practice.

We can now state the consistency result of the generalized spectral estimator:

**Theorem 5** (*Consistency*) *Under Assumptions 1-8, the generalized spectral estimator  $\widehat{\theta}$  is consistent for the model (4.1), i.e.,  $\widehat{\theta} \rightarrow \theta$  in probability.*

With two additional assumptions, it is possible to show that the generalized spectrum estimator is asymptotically Normally distributed. The estimator's asymptotic variance will be seen to depend on the Cramér-Von Mises criterion as well as the underlying data generating process.

**Assumption 4.9**  $\Sigma \triangleq \lim n \left( \frac{\partial D^2(\theta)}{\partial \theta} \Big|_{\theta=\theta_0} \right) \left( \frac{\partial D^2(\theta)}{\partial \theta'} \Big|_{\theta=\theta_0} \right)$  is a finite non-stochastic matrix.

The sample analog of  $\Sigma$  is

$$\widehat{\Sigma} = n \left( \frac{\partial D_n^2(\theta)}{\partial \theta} \Big|_{\theta=\widehat{\theta}} \right) \left( \frac{\partial D_n^2(\theta)}{\partial \theta'} \Big|_{\theta=\widehat{\theta}} \right)$$

where

$$\begin{aligned} \frac{\partial D_n^2(\theta)}{\partial \theta} \Big|_{\theta=\theta_0} &= \sum_{j=1}^{n-1} \left[ \frac{2(n-j)}{(j\pi)^2} \left( \sum_{t=j+1}^n \rho(Z_t, \theta) - \frac{1}{n-j} \sum_{t=j+1}^n \rho(Z_t, \theta) \right) \right. \\ &\quad \times \left( \sum_{s=j+1}^n \frac{\partial \rho(Z_s, \theta)}{\partial \theta} - \frac{1}{n-j} \sum_{s=j+1}^n \frac{\partial \rho(Z_s, \theta)}{\partial \theta} \right) \int_R \cos u(X_{t-j} - X_{s-j}) W(du) \Big|_{\theta=\widehat{\theta}} \end{aligned}$$

**Assumption 4.10**  $\rho(Z_t, \theta)$  is twice continuously differentiable in  $\theta$ .

**Theorem 6** (Asymptotic Normality) Under Assumptions 1-9,

$$\sqrt{n}(\widehat{\theta}_n - \theta) \rightarrow^d N(0, H(\theta_0)^{-1} \Sigma H(\theta_0)^{-1})$$

where  $H(\theta_0)$  is the Hessian of the minimand  $D^2$ . And the sample analog of  $H(\theta_0)$  is

$$\widehat{H}(\theta)$$

$$\widehat{H}(\theta)$$

$$\begin{aligned} &= \sum_{j=1}^{n-1} \left\{ \frac{2(n-j)}{(j\pi)^2} \left[ \left( \sum_{t=j+1}^n \frac{\partial \rho(Z_t, \theta)}{\partial \theta'} - \frac{1}{n-j} \sum_{t=j+1}^n \frac{\partial \rho(Z_t, \theta)}{\partial \theta'} \right) \left( \sum_{s=j+1}^n \frac{\partial \rho(Z_s, \theta)}{\partial \theta} - \frac{1}{n-j} \sum_{s=j+1}^n \frac{\partial \rho(Z_s, \theta)}{\partial \theta} \right) \right. \right. \\ &\quad + \left. \left( \sum_{t=j+1}^n \rho(Z_t, \theta) - \frac{1}{n-j} \sum_{t=j+1}^n \rho(Z_t, \theta) \right) \left( \sum_{s=j+1}^n \frac{\partial^2 \rho(Z_s, \theta)}{\partial \theta \theta'} - \frac{1}{n-j} \sum_{s=j+1}^n \frac{\partial^2 \rho(Z_s, \theta)}{\partial \theta \theta'} \right) \right] \\ &\quad \left. \int_R \cos u(X_{t-j} - X_{s-j}) W(du) \right\} \Big|_{\theta=\widehat{\theta}} \end{aligned}$$

The generalized spectral estimator  $\widehat{\theta}$  is consistent but inefficient. Following Newey(1990,1993) and Domínguez and Lobato(2004), an efficient estimator can

be obtained from the consistent estimators  $\widehat{\theta}$  via an additional Newton-Raphson step. It is computed as

$$\widehat{\theta}^E = \widehat{\theta} - \left[ \nabla_{\theta\theta'} D_n^2(\widehat{\theta}) \right]^{-1} \nabla_{\theta} D_n^2(\widehat{\theta})$$

where  $\nabla_{\theta\theta'} D_n^2(\widehat{\theta})$  and  $\nabla_{\theta} D_n^2(\widehat{\theta})$  are the gradient vector and Hessian matrix respectively both evaluated at the consistent estimator  $\widehat{\theta}$ . In the general case, both  $\nabla_{\theta\theta'} D_n^2(\widehat{\theta})$  and  $\nabla_{\theta} D_n^2(\widehat{\theta})$  involve estimating some conditional expectations which can usually be estimated based on kernels, series expansions, or nearest neighbors methods; see Newey (1990, 1993) and Robinson (1991). Although theoretically the asymptotic distribution is achieved after the first iteration, in practice carrying out additional iterations may improve the finite sample performance.

### 4.3 Simulation Studies

#### 4.3.1 Simulation Design

In this section, simulation studies will be conducted to check the finite sample performance of our proposed generalized spectral estimator(GSE). Comparisons will be made to Domínguez and Lobato's(2004) estimator(DL) and Berkowitz's(2001) power spectral estimator(PSE)<sup>3</sup>, respectively focused on Examp-1 which mainly illustrates the infinite dimensional conditioning information set and Example-2 which concerns the conditional identification prob-

---

<sup>3</sup>Berkowitz's(2001) estimator is originally named generalized spectral estimator. However, it is essentially depending on the linear dependence measure, power spectrum. Therefore, we re-label it as PSE to differentiate from our GSE which is indeed depending on Hong's(1999) generalized power spectrum and which inherits the true "general spectral" property.

lem. Furthermore, a third example defined by conditional moment restrictions with only one conditioning variable is considered for which both DL and our GSE are consistent.

**Example 3** Consider the following simple regression model:

$$Y_t = \alpha_0 X_t + \epsilon_t$$

where  $E[\epsilon_t|X_t] = 0$  and  $\epsilon_t \sim i.i.d.N(0, 1)$ . Equivalently this model can be identified by  $E[Y_t|X_t] = \alpha_0 X_t$ .

The objective of studying Example-3 is to investigate the efficiency of our estimator since, as discussed earlier, efficiency of the estimator may be sacrificed by incorporating the infinite dimensional conditioning information set completely to be free of the identification problem. In this example, both DL and GSE are consistent and hence the comparisons would show the efficiency loss of our estimator due to the redundant moment conditions.

We next briefly discuss the implementation of DL and PSE estimators. The DL estimator is constructed by the following conditional moment restriction:

$$E(\rho(Z_t, \theta_0)|X_t) = 0 \tag{4.15}$$

where only one single conditioning variable is in the conditional information set. From Billingsley (1995, Theorem 16.10iii), (4.15) is equivalent to

$$H(\theta, x) = 0$$

for almost all  $x \in R^q$ , where  $H(\theta, x) = E(\rho(Z_t, \theta)I(X_t < x))$ . Hence the uniqueness of the true parameter follows by

$$\int H(\theta, x)^2 dP_{X_t}(x) = 0 \iff \theta = \theta_0$$

based on which DL's estimator is constructed as:

$$\widehat{\theta}_{DL} = \arg \min_{\theta} \frac{1}{n^3} \sum_{l=1}^T \left( \sum_{t=1}^T \epsilon_t(\theta) I(X_t < X_l) \right)^2 \quad (4.16)$$

which is the sample analog of

$$\theta = \arg \min_{\theta} \int H(\theta, x)^2 dP_{X_t}(x)$$

The model for PSE is actually the general model we consider in (4.1),  $E[\rho(Z_t, \theta_0) | \mathcal{F}_{t-1}] = 0$ . However, only the implied necessary conditions represented by the unconditional moment conditions,  $E[\rho(Z_t, \theta_0) \rho(Z_{t-j}, \theta_0)] = 0$  *a.s.* for  $j \geq 1$ , are considered. Then the associated spectral density  $f_{PSE}(\omega; \theta) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} \eta_j(\theta) e^{-ij\omega}$ , where  $\eta_j(\theta)$  are the autocovariance at lag  $j$  for the sequence  $\{\rho(Z_{t-j}, \theta)\}_{j \geq 0}$ . When  $j = 0$ ,  $\eta_0(\theta) = \sigma^2$  is the variance of  $\rho(Z_t, \theta_0)$  and the spectral density becomes a flat spectrum, i.e.,  $f_{PSE}^0(\omega; \theta) = \frac{\eta_0(\theta)}{2\pi} = \frac{\sigma^2}{2\pi}$ . Define

$$U(\lambda; \theta) = \int_0^{\pi\lambda} (f_{PSE}(\omega; \theta) / \sigma^2 - \frac{1}{\pi}) d\omega$$

with its sample analog  $\widehat{U}(\lambda; \theta) = \int_0^{\pi\lambda} (\widehat{f}_{PSE}(\omega; \theta) / \widehat{\sigma}^2(\theta) - \frac{1}{\pi}) d\omega$ ,  $\forall \lambda \in (0, 1)$ . Motivated by the Cramér-Von Mises (CVM) test statistic in Durlauf (1991), the PSE is constructed as

$$\widehat{\theta}_{PSE} = \arg \min_{\theta} \int_0^1 \widehat{U}(\lambda; \theta)^2 d\lambda \quad (4.17)$$

$$= \arg \min_{\theta} \sum_{j=1}^{T-1} \left(1 - \frac{j}{T}\right) \frac{1}{(\pi j)^2} \left( \frac{\widehat{\eta}_j(\theta)}{\widehat{\sigma}^2(\theta)} \right)^2 \quad (4.18)$$

### 4.3.2 Simulation Evidence

First for Example-1, our GSE  $(\widehat{\alpha}_{1,GSE}, \widehat{\alpha}_{2,GSE})$  is compared with DL  $(\widehat{\alpha}_{1,DL}, \widehat{\alpha}_{2,DL})$ . As discussed in Section 4.1, DL in (4.16) which only considers one conditioning

Table 4.1: GSE vs DL for Example-1

	GSE		DL	
	True Values: $(\alpha_{10}, \alpha_{20})=(0.3333, 0.1667)$			
T	Bias	RMSE	Bias	RMSE
50	(0.0015, 0.0081)	(0.0203, 0.1424)	(0.0118, -0.0090)	(0.7663, 0.9351)
200	(0.0003, 0.0008)	(0.0035, 0.0604)	(-0.0114, 0.0184)	(0.7310, 0.8986)
400	(-0.0001,0.0005)	(0.0018, 0.0395)	(-0.0043, 0.0090)	(0.7384, 0.9198)

Notes: Simulation results for Example-1:  $Y_t = \alpha_1 X_t + \alpha_2 X_{t-1} + \epsilon_t$  with  $E(\epsilon_t | \mathcal{F}_t) = 0$ ,  $(\alpha_{10}, \alpha_{20}) = (1/3, 1/6)$ , and  $\beta_0 = 0.8$ . The number of replications is 500 and sample sizes considered are  $T = 50, 200$  and 400 respectively. Both the bias and RMSE are reported.

variable is not consistent while GSE, by incorporating the infinite dimensional conditional information set, can estimate the parameters consistently. The number of replications is 500 in all simulations and we consider three sample sizes  $T = 50, 200, 400$  respectively. The simulation results are presented in Table I, with both the mean and root mean squared error(RMSE) reported.

From Table 4.1, we can see that our GSE has excellent finite sample performance with the bias equal to (0.0015, 0.0081) even when the sample size is as small as 50 and the performance is indeed improving with the increase of the sample size. In contrast, DL's bias is relatively larger especially for  $\alpha_{20}$ ; the bias is as big as 0.0090 with the big sample size  $T=400$ . In addition, the bias performance of DL for  $\alpha_{20}$  is becoming worse and worse as we increase the sample size from  $T=50$  to  $T=400$ . And more mysteriously, our GSE's RMSE is far smaller than those of DL which are spuriously big, contrary to our theoretical intuition.

To explore the reason, we plot in Figure 4.1 the objective functions of both GSE and DL for a simulated sequence. By observing Figure 4.1 (b), we can see that the minimizers of DL objective function almost form a straight line, with an infinite number of points. In contrast, for our GSE there is a unique minimiza-

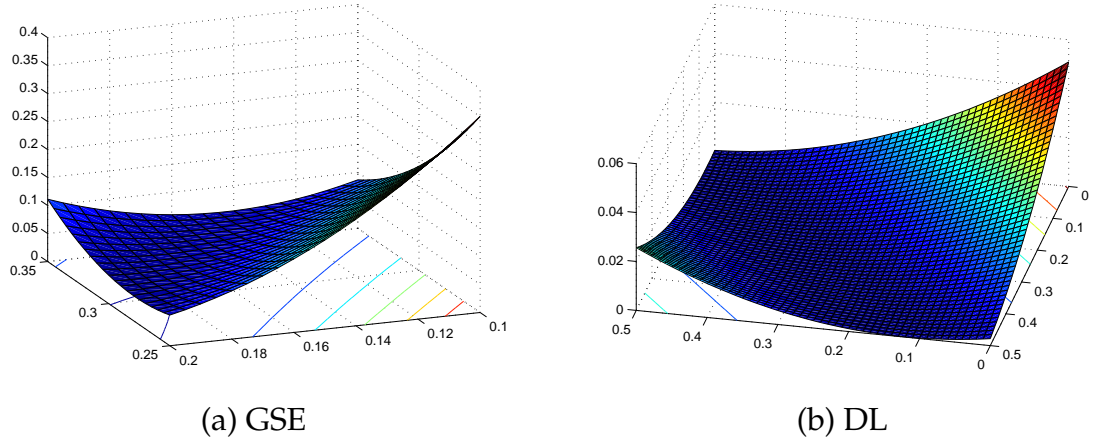


Figure 4.1: Objective Functions of GSE and DL for Example 1

Figure 4.1 (a) and (b) depict the objective functions of GSE and DL for the simulated sequence. GSE's objective function seems to have a unique minimum while the minimizers of DL objective function almost form a straight line, with an infinite number of points.

tion point due to the concavity although the function is very smooth around the minimization point. Therefore, the spurious performance of DL, the deteriorating bias performance with the increase of the sample size and the much bigger RMSE than our GSE contrary to the intuition, are in fact caused by the identification problem due to the in-efficient use of the conditional information set as discussed in Section 4.1. To further confirm this, we plot in Figure 4.2 both GSE and DL estimates for 10 replicated simulations. It can be seen easily that the DL estimates  $(\hat{\alpha}_{1,DL}, \hat{\alpha}_{2,DL})$  lie on a straight line approximately while the GSE  $\hat{\alpha}_{1,GSE}$  is almost fixed around the true value  $\alpha_{1_0}=0.3333$  and  $\hat{\alpha}_{2,GSE}$  is distributed around the true value with small variations. Therefore, the seemingly consistency of DL is actually caused by the feature that in simulations, the DL estimates from different replications are distributed around the true value pretty evenly and the calculated averages are closed to the true parameter values. Now both the deteriorating bias performance with the increase of the sample size and the spuriously bigger RMSE of DL than our GSE are within our expectation. They



are all evidences of the identification problem caused by the partial utilization of the full conditional information set, which leads to inconsistency of the DL estimators.

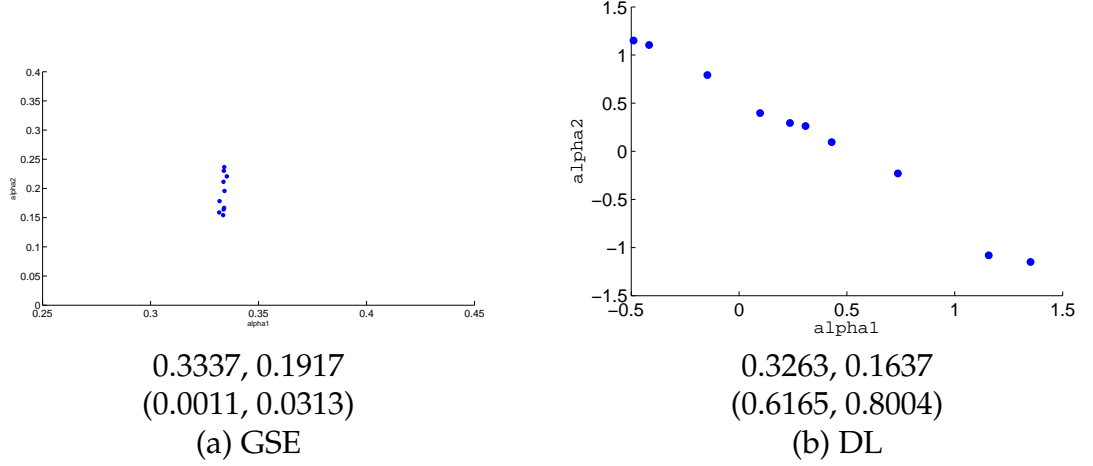


Figure 4.2: Scatters of GSE and DL for Example-1

The scatters of 10 replicated simulations for GSE and DL. The means(numbers in the first upper part) and their RMSEs(numbers reported in the lower parenthesis). The GSE ( $\hat{\alpha}_{1,GSE}, \hat{\alpha}_{2,GSE}$ ) are distributed around the true value with very small deviations but ( $\hat{\alpha}_{1,DL}, \hat{\alpha}_{2,DL}$ ) are scattered approximately on a straight line evenly.

Next for Example-2, our GSE  $\hat{\theta}_{GSE}$  is compared with the PSE  $\hat{\theta}_{PSE}$ . As discussed in Section 4.1, PSE in (4.17) which is based on the linear dependence measure–power spectrum–is not consistent while GSE, via the nonlinear dependence measure–generalized spectrum proposed in Hong(1999)–can estimate  $\theta$  consistently. Example-2 is an AR(1) model  $X_t = \theta X_{t-1} + \epsilon_t$  with  $E[\epsilon_t | \mathcal{F}_{t-1}] = 0$  and we assume  $\theta_0 = 0.5$  and  $\epsilon_t \sim i.i.d.N(0, 1)$ . The number of replications is 500 in all simulations and we consider three sample sizes  $T = 50, 200, 400$  respectively. The simulation results are presented in Table 4.2, with both the mean and root mean squared error(RMSE) reported.

We can see from Table 4.2 that our estimator  $\hat{\theta}_{GSE}$  has excellent finite sample performance in terms of both the bias and RMSE, which are as small as -0.0045

Table 4.2: GSE vs PSE for Example-2

	GSE		PSE	
	True Values: $\theta_0 = 0.5$			
T	Bias	RMSE	Bias	RMSE
50	-0.0405	0.1446	1.5477	1.5529
200	-0.0106	0.0627	1.5505	1.5554
400	-0.0045	0.0475	1.5396	1.5432

Note: For Example 2,  $X_t = X_{t-1}^2 + \epsilon_t$  with  $E[\epsilon_t | \mathcal{F}_{t-1}] = 0$ .  $\theta_0 = 0.5$ , and  $\epsilon_t \sim i.i.d.N(0, 1)$ . The number of replications is 500 and sample sizes are  $T = 50, 200$  and  $400$  respectively. Both the bias and RMSE are reported.

and 0.0475 respectively. Moreover, with the sample size increasing, both the bias and RMSE are improving quickly. In contrast, it seems that the PSE  $\widehat{\theta}_{PSE}$  is not consistent at all with the bias as big as 1.5396. Intuitively, this is caused by the conditional identification problem as discussed in Section-2. To further gauge this explanation, we plot in Figure 4.3 the objective functions of both GSE and PSE. Obviously,  $\widehat{\theta}_{GSE}$  is the unique minimization point of the GSE objective function while for PSE, two local minimum points exist: one is the true value  $\theta_0 = 0.5$  and the other  $\theta_1 = 2$  due to the conditional identification problem. The simulation results show that  $\widehat{\theta}_{PSE}$  converge to the false parameter value.

Lastly for Example-3, our GSE  $\widehat{\alpha}_{GSE}$  is compared with DL  $\widehat{\alpha}_{DL}$  again. As discussed earlier, both GSE and DL are consistent estimators for this example. Such as comparison can enable us to investigate the efficiency of our estimator relative to other consistent estimators which utilize less moment conditions. The true parameter value is set at  $\alpha_0 = 1/3$ . The number of replications is 500 and we consider three sample sizes  $T = 50, 200, 400$  respectively. The simulation results are presented in Table 4.3, with both the mean and root mean squared

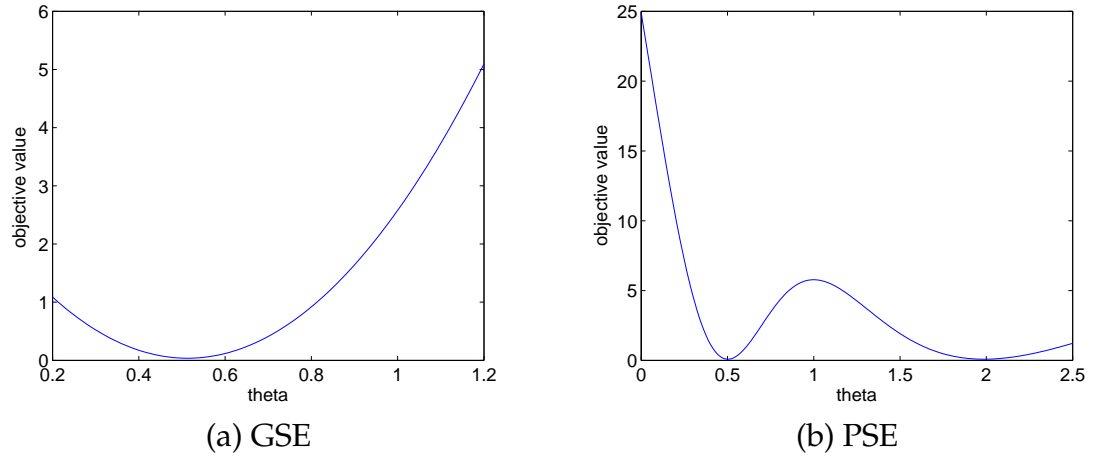


Figure 4.3: Objective Functions of GSE and PSE for Example-2

Figure 4.3 (a) is the objective function of GSE, and (b) that of the PSE. The former clearly has a unique minimum around the true parameter value while the latter has two local minimum points, one the true parameter and the other a value different from the true due to the conditional identification problem.

Table 4.3: GSE vs DL for Example-3

	GSE		DL	
	True Values: $\alpha_0 = 0.3333$			
T	Bias	RMSE	Bias	RMSE
50	-0.0019	0.0948	0.0058	0.0783
200	0.0025	0.0870	-0.0017	0.0376
400	0.0001	0.0850	-0.0006	0.0270

Notes: For Example-3,  $Y_t = \alpha_0 X_t + \epsilon_t$  where  $E[\epsilon_t | X_t] = 0$ ,  $\epsilon_t \sim i.i.d.N(0, 1)$  and  $\alpha_0 = 1/3$ . The number of replications is 500 and sample sizes are  $T = 50, 200$  and  $400$  respectively. Both the bias and RMSE are reported.

error(RMSE) reported.

As can be seen, both GSE and DL have excellent finite sample performance in terms of both bias and RMSE. There is no observable evidence of dominating performance in terms of bias while the RMSEs of PSE is truly smaller than those

of GSE. This confirms the theoretical conclusions that both GSE and DL are consistent estimators and efficiency of the GSE is indeed sacrificed by incorporating the infinite dimensional conditioning information set completely to be free of the identification problem. We also plot in Figure 4.4 the objective functions of both GSE and DL. It is very obvious that both  $\hat{\alpha}_{GSE}$  and  $\hat{\alpha}_{DL}$  are minimizing points around the true value  $\alpha_0 = 1/3$ . In fact, these results further confirm our earlier discussions about Example-1 since they show that as long as both estimators are consistent, the GSE would have a smaller efficiency than the DL, implying bigger RMSEs. Henceforth, the bigger RMSEs of GSE than those of DL in Example 3 are spurious in this sense and must be indicative of inconsistency.

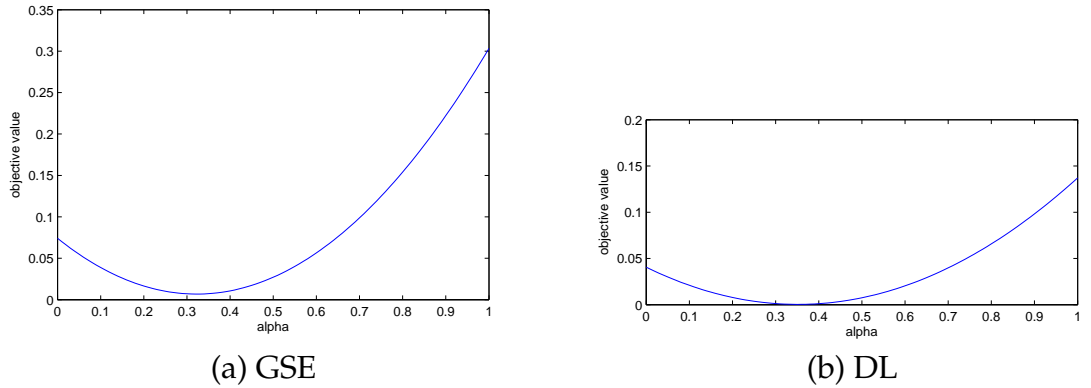


Figure 4.4: Objective Functions of GSE and DL for Example-3

Figure 4.4 (a) is the objective function of GSE, and (b) that of the DL. Both have the unique minimization points around the true value.

## 4.4 Empirical Application: Estimating the CCAPM

In this section, the proposed GSE will be applied to estimate the CCAPM which first appears in Lucas (1978) and Breeden (1979) and which has been investigated empirically innumerable times. It is found that the CCAPM is not consistent

with the observed data in reality and hence is not a great empirical success. The most interesting evidence casting doubt on the model, is the so-called risk-premium puzzle. Basically, it refers to the fact that a huge level of risk aversion, based on the calibration exercises of Mehra and Prescott (1985), is needed to match the equity premium observed for stock data while the estimated coefficient of risk aversion is pretty small relatively<sup>4</sup>.

Assume perfect asset markets and homogenous agents who maximize the expected time-separable utility of consumption, the rational expectation equation is

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^t U(C_t)$$

where  $\beta$  is a constant time discount factor,  $C_t$  is time- $t$  consumption and the expectation is conditioned on information available up to time  $t_0$ . The representative is assumed to display constant relative risk aversion, e.g.  $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$ . Along with the usual budget constraint, this implies the following econometric model based on the first-order conditions:

$$E[\beta R_t (\frac{C_t}{C_{t-1}})^{-\gamma} - 1 | \mathcal{F}_{t-1}] = 0 \quad (4.19)$$

where  $R_t$  is the one-period real return on unconsumed wealth and  $\gamma$  is the coefficients of related risk aversion.  $\beta$  and  $\gamma$  are the two parameters we are interested to estimate. To be consistent with the general framework in (4.1), we define  $\rho(Z_t, \theta) = \beta R_t (\frac{C_t}{C_{t-1}})^{-\gamma} - 1$  with  $Z_t = [R_t, \frac{C_t}{C_{t-1}}]'$  and  $\theta = [\beta, \gamma]$ .

Hansen and Singleton (1982) estimate (4.19) by a standard GMM procedure in the time domain. First assume  $\rho(Z_t, \theta_0)$  have a finite second moment and then

---

<sup>4</sup>There are many other studies leading to the inconsistency of the CCAPM and real data, including formal testing procedures of Hansen and Singleton (1982, 1983) which reject the model econometrically, and those analyzing the stochastic discount factors directly like Cochrane and Hansen (1992), Burnside (1994), and Cecchetti et al. (1994) showing that the CCAPM implies a stochastic discount factor that violates the Hansen–Jagannathan volatility bounds.

define the function  $f$  by

$$f(Z_t, X_t, \theta) = \rho(Z_t, \theta) \otimes X_{t,n}$$

where  $X_{t,n}$  is the vector of instruments formed using lagged values of  $Z_t$  belonging to the information set, i.e.,  $X_{t,n} = [Z_{t-1}, Z_{t-2}, \dots, Z_{t-n}]'$  is a  $2n$ -dimensional vector for the fixed  $n$  lags (Hansen and Singleton (1982) set  $n = 1, 2, 4$  and  $6$  respectively in their study). Their GMM estimator is based on the following unconditional moment restriction implied by the conditional moment conditions in (4.19):

$$E[f(Z_t, X_{t,n}, \theta_0)] = 0$$

with the sample analogue

$$g_T = \frac{1}{T-n} \sum_{t=1}^{T-n} f(Z_t, X_{t,n}, \theta)$$

which should be equal to 0 when evaluated at  $\theta = \theta_0$ . The GMM estimator can then be obtained as:

$$\widehat{\theta}_{GMM} = \arg \min g_T' W_T g_T(\theta) \quad (4.20)$$

where  $W_T$  is an  $2n$  by  $2n$  symmetric, positive definite weighting matrix.

The GMM estimates for  $\beta$  and  $\gamma$  of the CCAPM (4.19) are typically around 1 and 2 respectively. In contrast, the calibration exercises as those in Campbell et al. (1997) suggest that  $\gamma$  should be close to 20 for the model to support the equity risk-premium, which is a statement of the equity premium puzzle. However, as discussed in Section 4.1 and investigated in Section 4.3 by simulation studies, the GMM estimator in (4.20) which only employs a fixed and finite number of conditioning variables in the infinite dimensional conditional information set may not be consistent at all. It is very possible that the true parameter is much bigger than what GMM obtains.

In Berkowitz (2001), the PSE is also applied to estimate the CCAPM in (4.19) as follows:

$$\begin{aligned}\widehat{\theta}_{PSE} &= \arg \min \int_0^1 \widehat{U}(\lambda; \theta)^2 d\lambda \\ &= \arg \min \sum_{j=1}^{T-1} \left(1 - \frac{j}{T}\right) \frac{1}{(\pi j)^2} \left(\frac{\widehat{\eta}_j(\theta)}{\widehat{\sigma}^2(\theta)}\right)^2\end{aligned}\quad (4.21)$$

where  $\widehat{\eta}_j(\theta)$  is the sample autocovariance of  $\rho(Z_t, \theta)$  and  $\rho(Z_{t-j}, \theta)$  and  $\widehat{\sigma}^2(\theta)$  is the sample variance of  $\rho(Z_t, \theta)$ . The PSE estimate for  $\beta$  is still around 1 while that of  $\gamma$  is fairly bigger than the GMM estimate and reaches as large as 4.4349<sup>5</sup>. But according to the discussions in Section 2 and simulations in Section 4 again, the PSE  $\widehat{\theta}_{PSE}$  may not be consistent at all due to the conditional identification problem. The true parameter values could be very different from  $\widehat{\theta}_{PSE}$  which only incorporates the linear dependence in the process. The problem could be very serious considering the nonlinear features of the asset return dynamics.

Therefore, we shall estimate the CCAPM by the proposed GSE in this paper, which is truly consistent by incorporating the infinite dimensional conditioning set and nonlinear dependence structures simultaneously. The comparisons will be made to Berkowitz's(2001) PSE in (4.21) and the GMM estimator of Hansen and Singleton (1982) in (4.20). For the latter, a 2-step feasible procedure is employed to obtain the "optimal" GMM estimator. First, a preliminary GMM estimator  $\theta_T$  of  $\theta_0$  is obtained by initially using a suboptimal choice of  $W_T$ , the  $2n$  by  $2n$  identity matrix for instance. Then,  $\theta_T$  can be employed to calculate the optimal weighting matrix  $W_T^*$  as follows:

$$W_T^* = \left\{ R_T(0) + \sum_{j=1}^{n-1} (R_T(j) + R_T(j)') \right\}^{-1}$$

---

<sup>5</sup>Berkowitz(2001) also applies the PSE on a subset of frequencies and it is actually a special feature of his study. However, partial utilization of the frequencies will introduce an additional identification problem since it is the information on all frequencies that identify the parameters.

where  $R_T(j) = \frac{1}{T-n} \sum_{t=1+j}^{T-n} f(Z_t, X_{t,n}, \theta) f(Z_t, X_{t,n}, \theta)'$  when  $n = 1$ . When  $n > 1$ , Newey-West method (Newey and West, 1987) can be adopted to give a heteroskedasticity-autocorrelation consistent variance-covariance matrix, i.e.

$$W_T^* = \left\{ R_T(0) + \sum_{j=1}^{n-1} (1 - j/n)(R_T(j) + R_T(j)') \right\}^{-1}$$

Then the optimal  $\widehat{\theta}_{GMM}$  can be obtained by using  $W_T^*$  in (4.20). In addition, we take  $n = 6$  for our empirical application.

Moreover, our GSE can be calculated by taking  $W(\cdot)$  as a standard normal distribution in (4.14), which avoids the numerical integration:

$$\begin{aligned} \widehat{\theta}_{GSE} &= \arg \min_{\theta} D_n^2(\theta) \\ &= \arg \min_{\theta} \sum_{j=1}^{T-1} \frac{1}{T-j} \frac{1}{(j\pi)^2} \left( \sum_{t=j+1}^T \sum_{s=j+1}^T (\rho(Z_t, \theta) - \bar{\rho}_{T-j})(\rho(Z_s, \theta) - \bar{\rho}_{T-j}) \exp(-0.5(X_{t-j} - X_{s-j})^2) \right) \end{aligned}$$

where  $X_{t-j}$  is the  $j$ th lag of  $Z_t$ ,  $\bar{\rho}_{T-j} = \frac{1}{T-j} \sum_{s=j+1}^T \rho(Z_s, \theta)$ .

The time period of the data is from January 1959 to December 1972. The monthly seasonally adjusted observations on real personal consumption index of nondurables and service are obtained from Bureau of Economic Analysis NIPAs (2.8.3). Real per capita terms are constructed by dividing each observation by price deflator from NIPAs (2.8.4) and then the associated observation in population from NIPAs (2.6). Two different measures of consumption are considered: nondurables plus services (NDS) and nondurables (ND). Asset returns are constructed from the Center for Research in Security Prices (CRSP) equally-weighted NYSE index (EWR) and value-weighted NYSE index (VWR). Nominal returns are converted to real returns by dividing by the implicit deflator associated with the measure of consumption.

Table 4.4 presents the estimators and their associated standard errors of the



CCAPM for those three estimators we consider, GMM, PSE and GSE. We consider four combinations by measures of returns and consumptions. It can be seen that the three estimators of  $\beta$  are pretty similar, all close to 1 and all significant according to their standard errors. Another observation is that the GSE of  $\beta$ ,  $\widehat{\beta}_{GSE}$ , is the biggest among the three estimators and there are no big differences for different combinations of the consumption and asset return measures, which is true for all of the three estimators.

In contrast, for the parameter  $\gamma$ , there are substantive differences among the three estimators considered. It can be observed that our proposed GSE  $\widehat{\gamma}_{GSE}$  has the biggest value and  $\widehat{\gamma}_{PSE}$  is a bit larger than the smallest  $\widehat{\gamma}_{GMM}$ . Actually the range of the estimates for  $\gamma$  is 2.024 to 4.670 for GMM, 6.281 to 6.862 for PSE and 7.650 to 8.600 for GSE. According to the earlier discussions, the difference between  $\widehat{\gamma}_{GMM}$  and  $\widehat{\gamma}_{PSE}$  mainly reflects the extension of instruments for the fixed 6 lags to the whole conditional information set, and the gap between  $\widehat{\gamma}_{PSE}$  and  $\widehat{\gamma}_{GSE}$  is due to the capturing of nonlinear dependence in the latter, which is omitted in the latter. Another observation is that the combinations of consumption and return measures have some impacts on the estimation results: the GMM estimate is as big as 4.670 for VWR and NDS while only 2.024 for EWR and NDS; both PSE and GSE have bigger values for ND than for NDS.

Therefore, contrary to the intuition and estimation results of the existing estimators like GMM and PSE which assert a relatively small risk aversion to economic agents, the estimation evidence of our proposed GSE indicate a much larger coefficient of risk aversion. It can be justified intuitively by two thoughts about the risk attitudes of people: first, past information which happened a long time period ago, does affect economic agents' reactions to the risks in the market

Table 4.4: GMM, PSE and GSE for CCAPM

Cons	VWR			VWR			EWR			EWR		
Return	ND			NDS			ND			NDS		
Estimator	GMM	PSE	GSE	GMM	PSE	GSE	GMM	PSE	GSE	GMM	PSE	GSE
$\widehat{\gamma}$	2.356	6.862	8.600	4.670	6.308	7.875	2.448	6.861	8.438	2.024	6.281	7.650
$\widehat{SE}(\gamma)$	2.667	0.001	0.019	5.091	0.002	0.007	3.178	0.001	0.007	5.732	0.002	0.004
$\widehat{\beta}$	0.968	0.981	0.990	0.972	0.946	0.990	0.976	0.952	0.990	0.976	0.935	0.990
$\widehat{SE}(\beta)$	0.013	0.004	0.083	0.016	0.004	0.005	0.018	0.005	0.030	0.021	0.004	0.003

Notes: Empirical estimates of  $\beta$  and  $\gamma$  and associated standard errors for three estimators, GMM, PSE and GSE are reported. Two different measures of consumption are considered: nondurables plus services (NDS) and nondurables (ND) while two sequences of asset returns are constructed, one equally-weighted NYSE index (EWR) and the other value-weighted NYSE index (VWR).

even today; secondly, the reactions to these market risks are highly nonlinear, especially to those risks related to unexpected jumps which are not uncommon. Of course, although our GSE implies a much higher risk aversion than the existing methods, it still cannot account for the equity risk-premium completely since we need  $\gamma$  to be at least 20 to match it. Our point here is that it does represent a direction which can decrease the distance between the theoretical models and the empirical evidences. Our hope is that combined with new theoretical efforts<sup>6</sup>, our econometric methods could shed more lights on this puzzle.

## 4.5 Conclusion

In this paper, a generalized spectral estimator is proposed via frequency domain methods for a class of time series models defined by conditional moment

<sup>6</sup>For example, Campbell and Cochrane(1999) propose a model with habit persistence and Constantinides and Duffie(1996) focus on the uninsured idiosyncratic risks. See Chp. 21 of Cochrane(2001) for an excellent survey.

restrictions with infinite dimensional conditioning set. The framework is general enough to cover most models which can be represented by conditional moment restrictions as special cases, including IV, nonlinear dynamic regression models, and rational expectations models such as consumption based asset pricing models (CCAPM). The estimator is obtained by minimizing the Cramér-Von Mises distance between the unrestricted generalized spectral distribution function and the model-implied correspondent, which is equivalent to setting a pairwise nonlinear dependence measure as close as possible to zero for each time lag order. It can be understood as a GMM estimator based on a set of moment conditions which grow with sample size. Not only is the infinite dimensional conditioning information embedded in this estimator, but also the nonlinear dependence is captured. Another feature is the simplicity since its implementation does not require selecting any user-chosen number. Simulation studies show that unlike existing estimators which can only deal with either linear dependence or a fixed finite number of conditioning variables separately instead of simultaneously, our proposed estimator are free of any identification problem as expected by incorporating both nonlinear dependence and infinite dimensional conditioning information. An empirical application for estimating CCAPM is conducted and we find that economic agents are much more risk-averse according to our estimator than what Hansen and Singleton's (1982) GMM estimation results imply.

## APPENDIX A

### APPENDIX OF CHAPTER 2

#### A.1 Figures and Tables

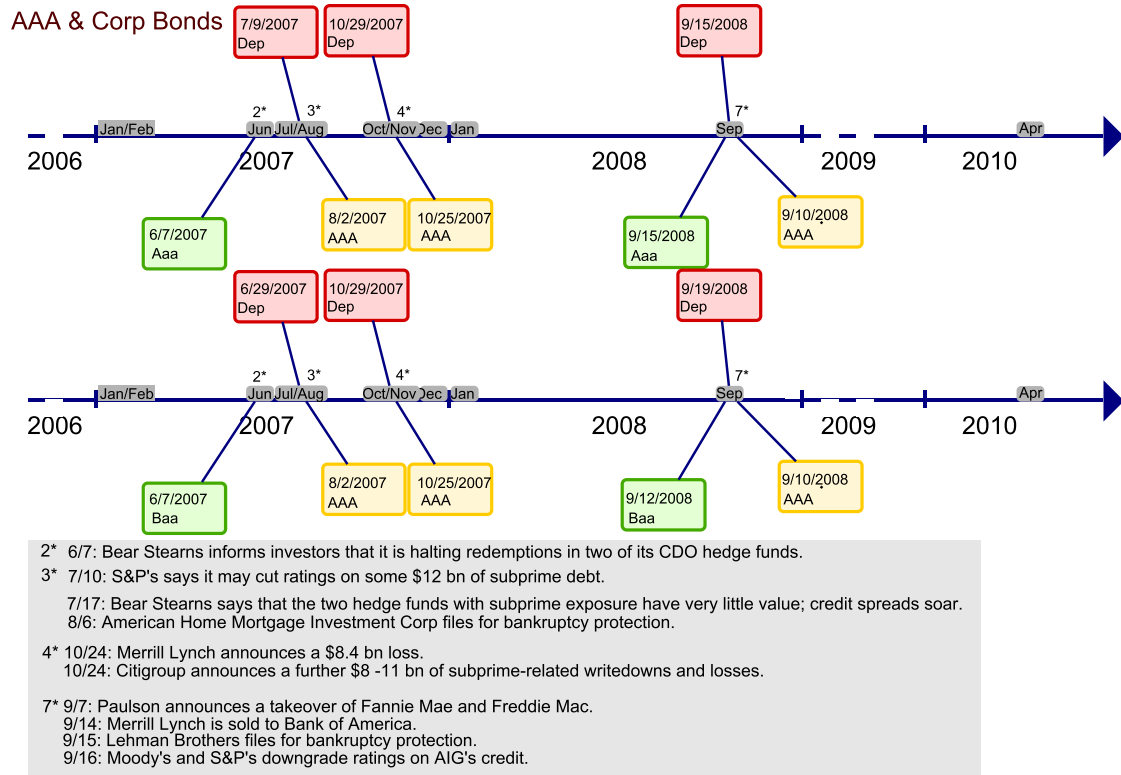


Figure A.1: Detected Abrupt Changes Between ABX AAA and Corporate Bonds

This figure shows the time lines of the abrupt changes between the AAA index and the corporate bonds (Moody's Aaa and Baa) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the AAA and the corporate bonds. The green frames indicate the dates when the time series of the corporate bonds change abruptly. The yellow frames correspond to abrupt changes in the AAA.

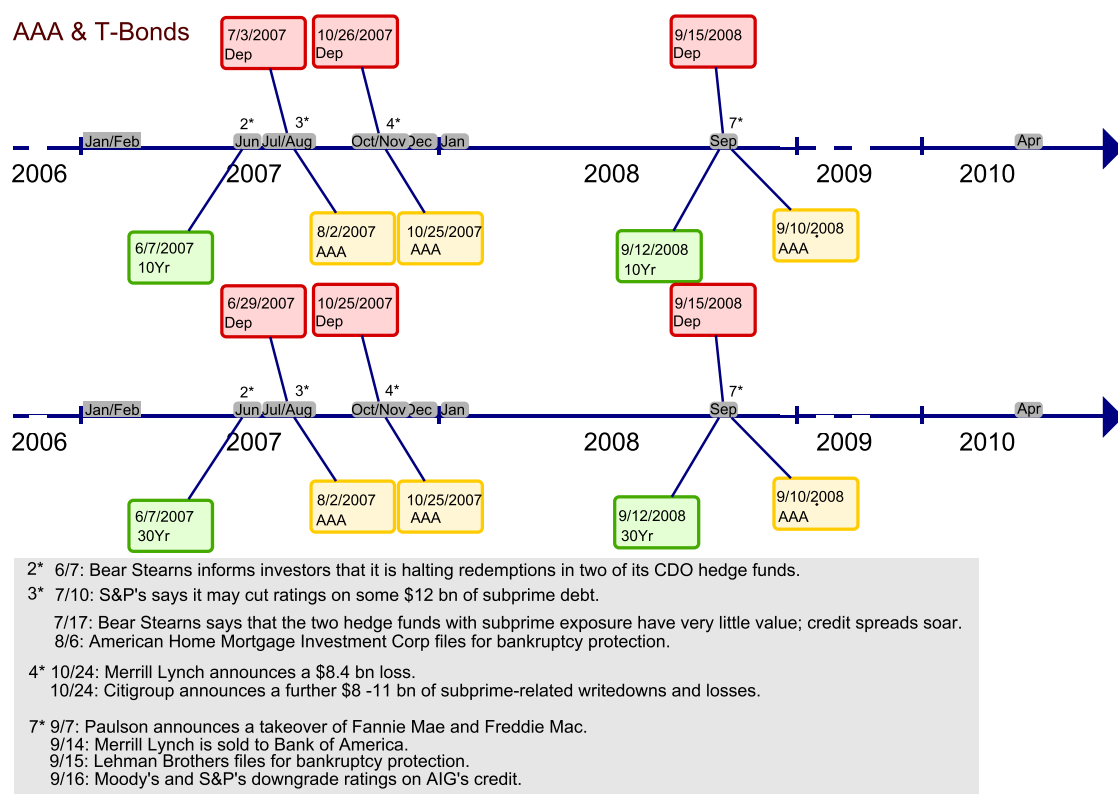
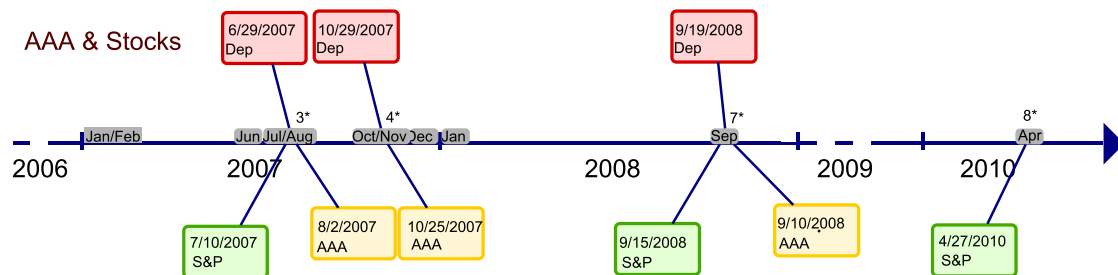


Figure A.2: Detected Abrupt Changes Between ABX AAA and Treasury Bonds

This figure shows the time lines of the abrupt changes between the AAA index and the Treasury bonds (10 year's and 30 years) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the AAA and the Treasury bonds. The green frames indicate the dates when the time series of the Treasury bonds change abruptly. The yellow frames correspond to abrupt changes in the AAA.



- 3\* 7/10: S&P's says it may cut ratings on some \$12 bn of subprime debt.  
 7/17: Bear Stearns says that the two hedge funds with subprime exposure have very little value; credit spreads soar.  
 8/6: American Home Mortgage Investment Corp files for bankruptcy protection.
- 4\* 10/24: Merrill Lynch announces a \$8.4 bn loss.  
 10/24: Citigroup announces a further \$8 -11 bn of subprime-related writedowns and losses.
- 7\* 9/7: Paulson announces a takeover of Fannie Mae and Freddie Mac.  
 9/14: Merrill Lynch is sold to Bank of America.  
 9/15: Lehman Brothers files for bankruptcy protection.  
 9/16: Moody's and S&P's downgrade ratings on AIG's credit.
- 8\* 4/27: Goldman Sachs defends itself against allegations that it profited from the housing market.  
 4/27: S&P's cuts Greece and Portugal's debt ratings.

Figure A.3: Detected Abrupt Changes Between ABX AAA and S&P 500

This figure shows the time line of the abrupt changes between the AAA index and the S&P 500 Index and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the AAA and the S&P index. The green frames indicate the dates when the time series of the S&P index changes abruptly. The yellow frames correspond to abrupt changes in the AAA.

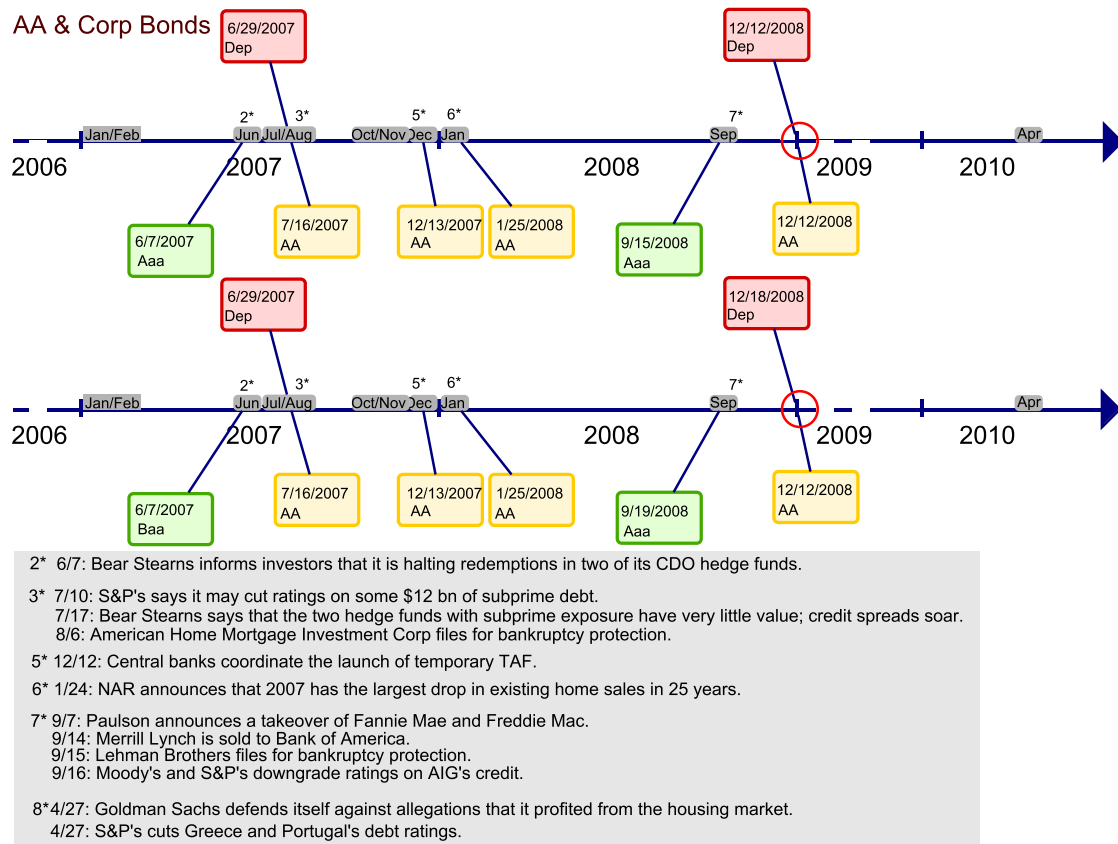


Figure A.4: Detected Abrupt Changes Between ABX-AA and Corporate Bonds

This figure shows the time lines of the abrupt changes between the AA index and the corporate bonds (Moody's Aaa and Baa) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the AA and the corporate bonds. The green frames indicate the dates when the time series of the corporate bonds change abruptly. The yellow frames correspond to abrupt changes in the AA.

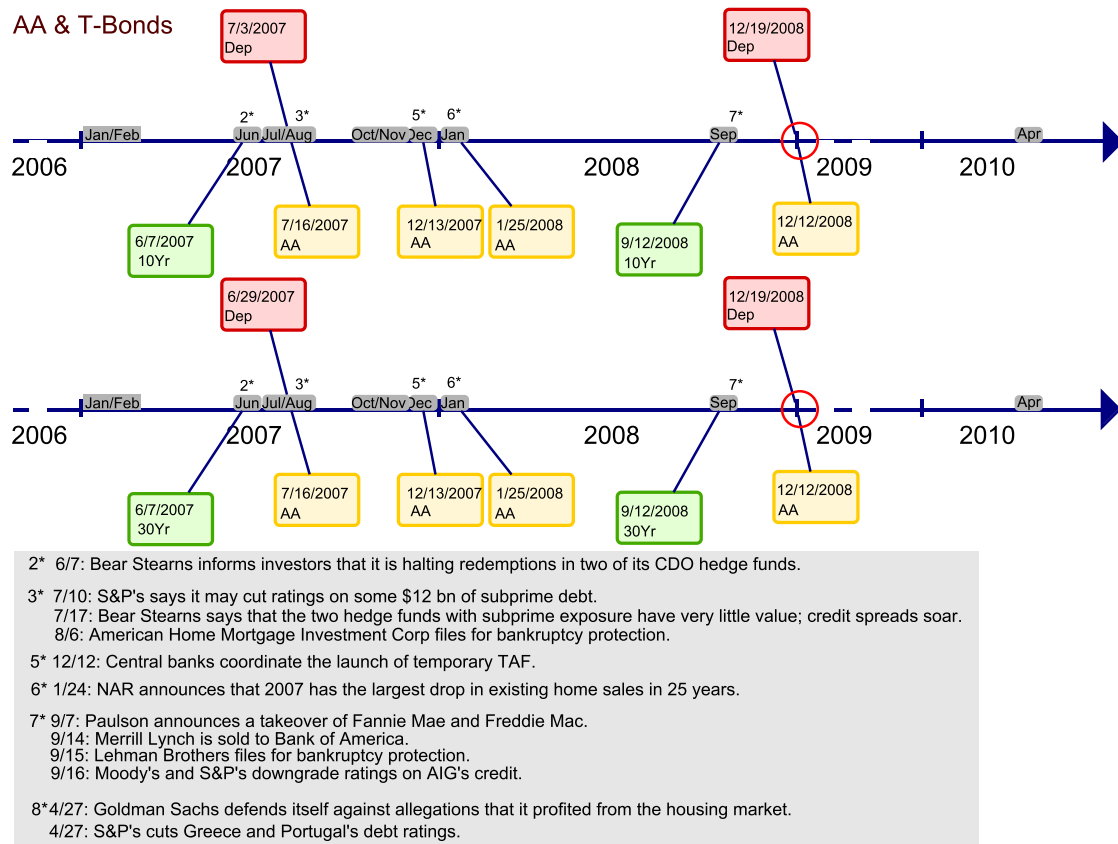
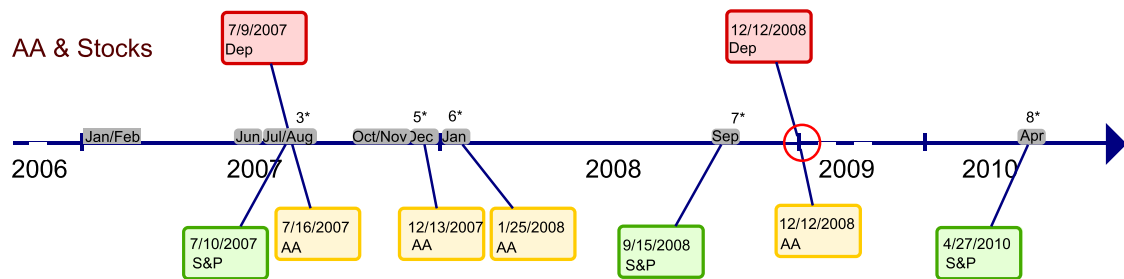


Figure A.5: Detected Abrupt Changes Between ABX-AA and Treasury Bonds

This figure shows the time lines of the abrupt changes between the AA index and the Treasury bonds (10 year's and 30 years) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the AA and the Treasury bonds. The green frames indicate the dates when the time series of the Treasury bonds change abruptly. The yellow frames correspond to abrupt changes in the AA.





- 3\* 7/10: S&P's says it may cut ratings on some \$12 bn of subprime debt.  
 7/17: Bear Stearns says that the two hedge funds with subprime exposure have very little value; credit spreads soar.  
 8/6: American Home Mortgage Investment Corp files for bankruptcy protection.
- 5\* 12/12: Central banks coordinate the launch of temporary TAF.
- 6\* 1/24: NAR announces that 2007 has the largest drop in existing home sales in 25 years.
- 7\* 9/7: Paulson announces a takeover of Fannie Mae and Freddie Mac.  
 9/14: Merrill Lynch is sold to Bank of America.  
 9/15: Lehman Brothers files for bankruptcy protection.  
 9/16: Moody's and S&P's downgrade ratings on AIG's credit.
- 8\* 4/27: Goldman Sachs defends itself against allegations that it profited from the housing market.  
 4/27: S&P's cuts Greece and Portugal's debt ratings.

Figure A.6: Detected Abrupt Changes Between ABX<sub>AA</sub> and S&P 500

This figure shows the time line of the abrupt changes between the AAA index and the S&P 500 Index and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the AA and the S&P index. The green frames indicate the dates when the time series of the S&P index changes abruptly. The yellow frames correspond to abrupt changes in the AA.

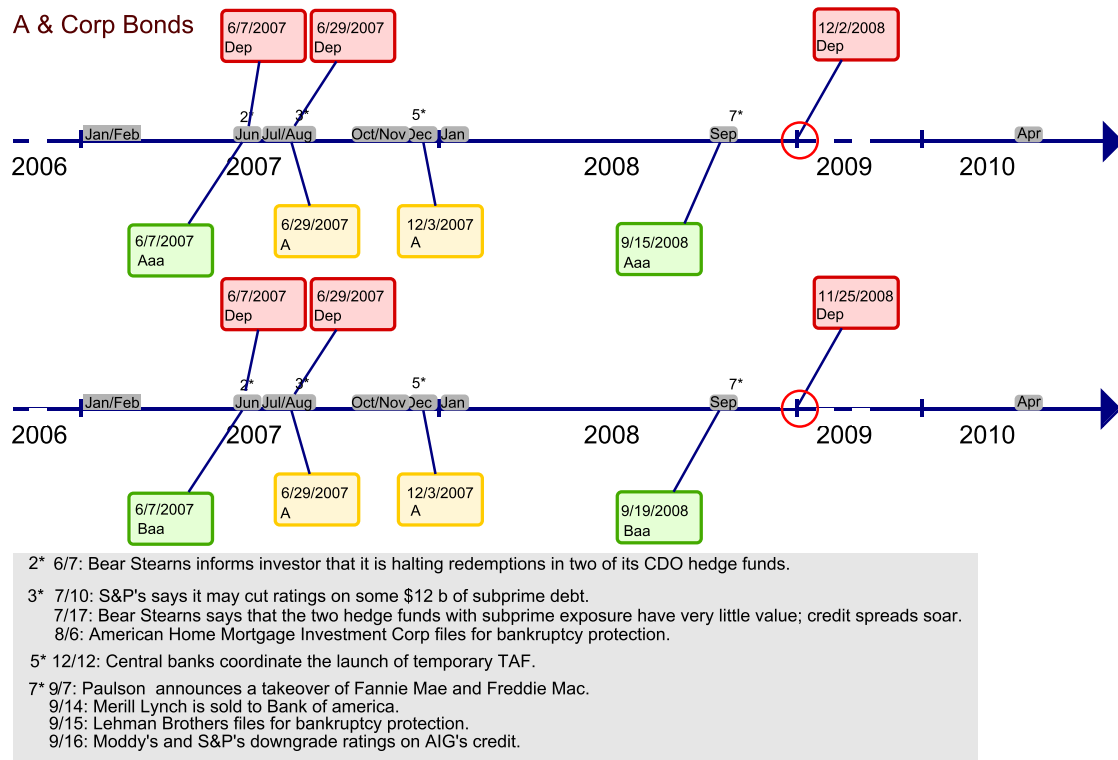


Figure A.7: Detected Abrupt Changes Between ABX\_A and Corporate Bonds

This figure shows the time lines of the abrupt changes between the A index and the corporate bonds (Moody's Aaa and Baa) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the A and the corporate bonds. The green frames indicate the dates when the time series of the corporate bonds change abruptly. The yellow frames correspond to abrupt changes in the A.

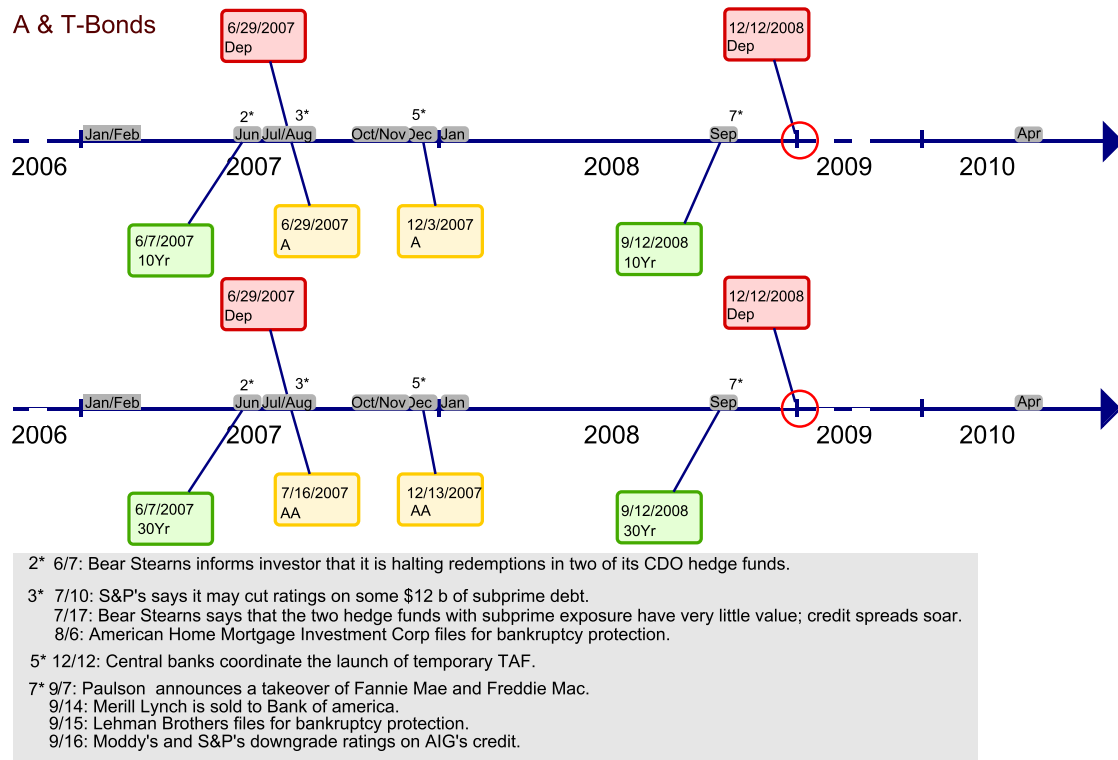
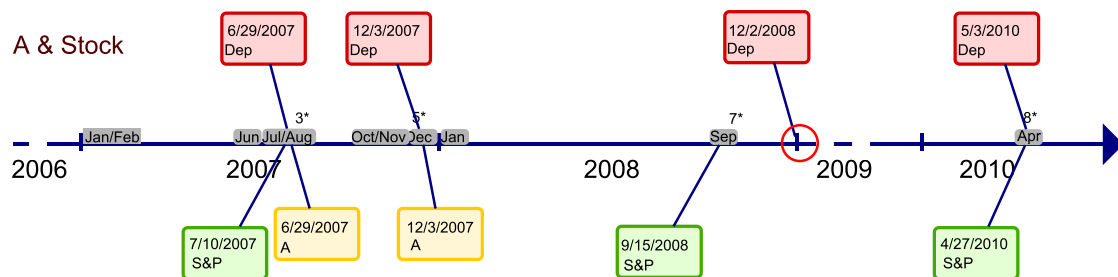


Figure A.8: Detected Abrupt Changes Between ABX\_A and Treasury Bonds

This figure shows the time lines of the abrupt changes between the A index and the Treasury bonds (10 year's and 30 years) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the A and the Treasury bonds. The green frames indicate the dates when the time series of the Treasury bonds change abruptly. The yellow frames correspond to abrupt changes in the A.



3\*7/10: S&P's says it may cut ratings on some \$12 b of subprime debt.  
 7/17: Bear Stearns says that the two hedge funds with subprime exposure have very little value; credit spreads soar.  
 8/6: American Home Mortgage Investment Corp files for bankruptcy protection.

5\*12/12: Central banks coordinate the launch of temporary TAF.

7\*9/7: Paulson announces a takeover of Fannie Mae and Freddie Mac.  
 9/14: Merrill Lynch is sold to Bank of America.  
 9/15: Lehman Brothers files for bankruptcy protection.  
 9/16: Moody's and S&P's downgrade ratings on AIG's credit.

8\*4/27: Goldman Sachs defends itself against allegations that it profited from the housing market.  
 4/27: S&P's cuts Greece and Portugal's debt ratings.

Figure A.9: Detected Abrupt Changes Between ABX\_A and S&P 500

This figure shows the time line of the abrupt changes between the A index and the S&P 500 Index and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the A and the S&P index. The green frames indicate the dates when the time series of the S&P index changes abruptly. The yellow frames correspond to abrupt changes in the A.

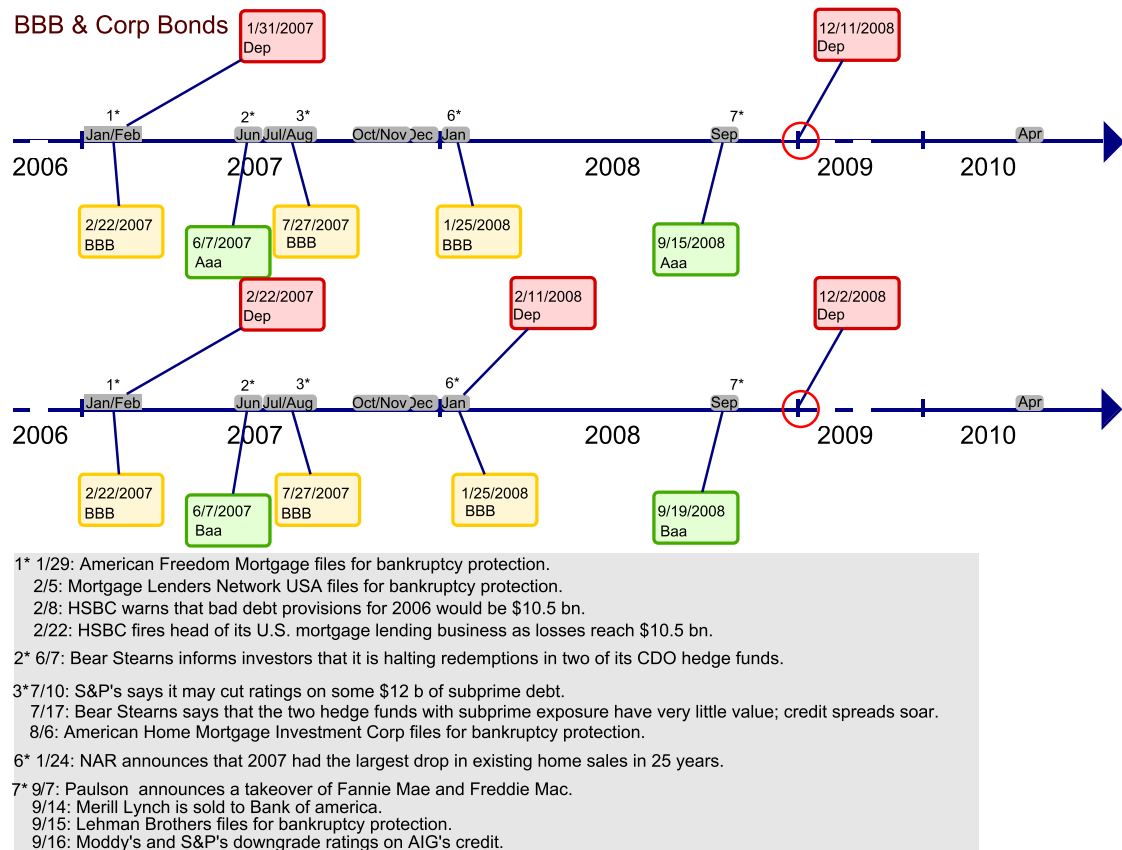


Figure A.10: Detected Abrupt Changes Between ABX.BBB and Corporate Bonds

This figure shows the time lines of the abrupt changes between the BBB index and the corporate bonds (Moody's Aaa and Baa) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the BBB and the corporate bonds. The green frames indicate the dates when the time series of the corporate bonds change abruptly. The yellow frames correspond to abrupt changes in the BBB.

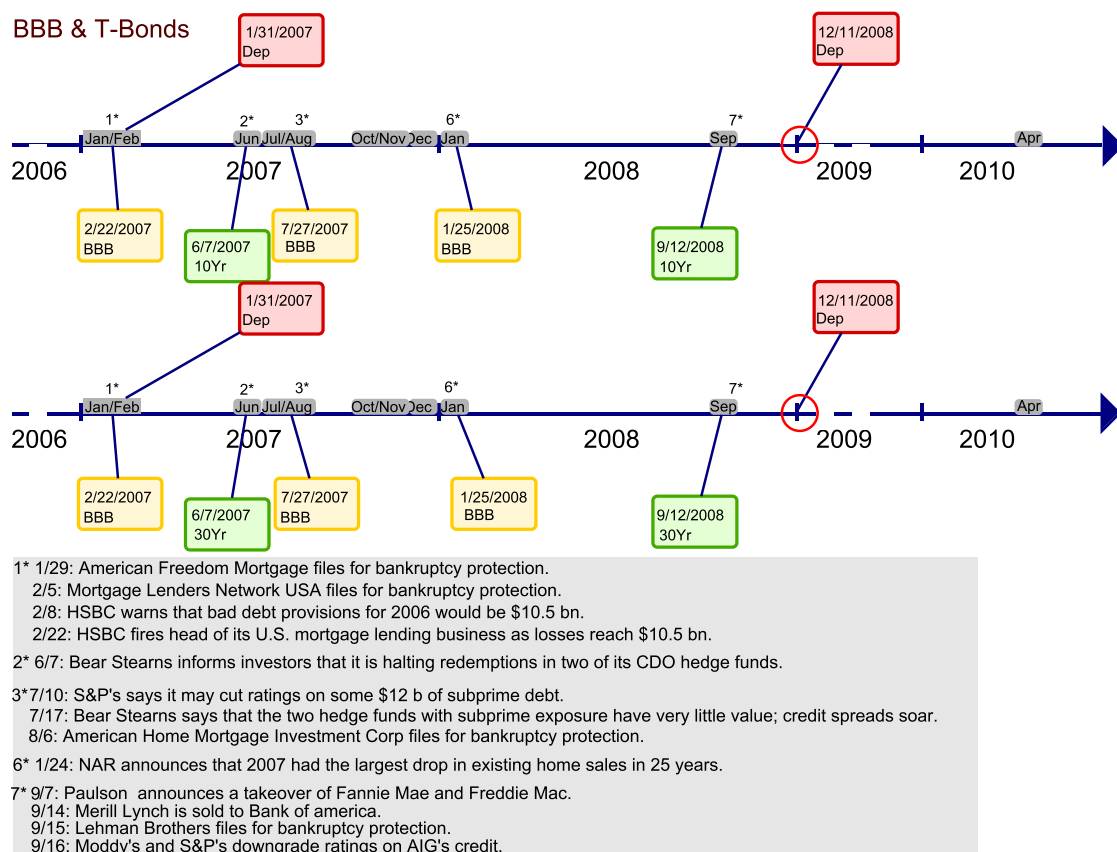
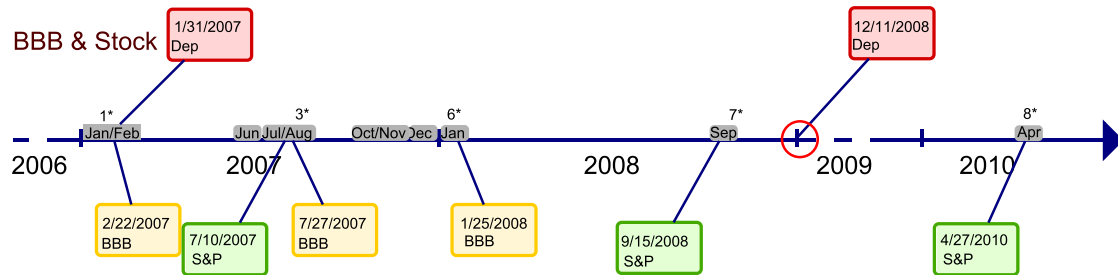


Figure A.11: Detected Abrupt Changes Between ABX\_BBB and Treasury Bonds

This figure shows the time lines of the abrupt changes between the BBB index and the Treasury bonds (10 year's and 30 years) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the BBB and the Treasury bonds. The green frames indicate the dates when the time series of the Treasury bonds change abruptly. The yellow frames correspond to abrupt changes in the BBB.



- 1\* 1/29: American Freedom Mortgage files for bankruptcy protection.
- 2/5: Mortgage Lenders Network USA files for bankruptcy protection.
- 2/8: HSBC warns that bad debt provisions for 2006 would be \$10.5 bn.
- 2/22: HSBC fires head of its U.S. mortgage lending business as losses reach \$10.5 bn.
- 3\* 7/10: S&P's says it may cut ratings on some \$12 b of subprime debt.
- 7/17: Bear Stearns says that the two hedge funds with subprime exposure have very little value; credit spreads soar.
- 8/6: American Home Mortgage Investment Corp files for bankruptcy protection.
- 6\* 1/24: NAR announces that 2007 had the largest drop in existing home sales in 25 years.
- 7\* 9/7: Paulson announces a takeover of Fannie Mae and Freddie Mac.
- 9/14: Merrill Lynch is sold to Bank of America.
- 9/15: Lehman Brothers files for bankruptcy protection.
- 9/16: Moody's and S&P's downgrade ratings on AIG's credit.
- 8\* 4/27: Goldman Sachs defends itself against allegations that it profited from the housing market.
- 4/27: S&P's cuts Greece and Portugal's debt ratings.

Figure A.12: Detected Abrupt Changes Between ABX\_BBB and S&P 500

This figure shows the time line of the abrupt changes between the BBB index and the S&P 500 Index and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the BBB and the S&P index. The green frames indicate the dates when the time series of the S&P index changes abruptly. The yellow frames correspond to abrupt changes in the BBB.

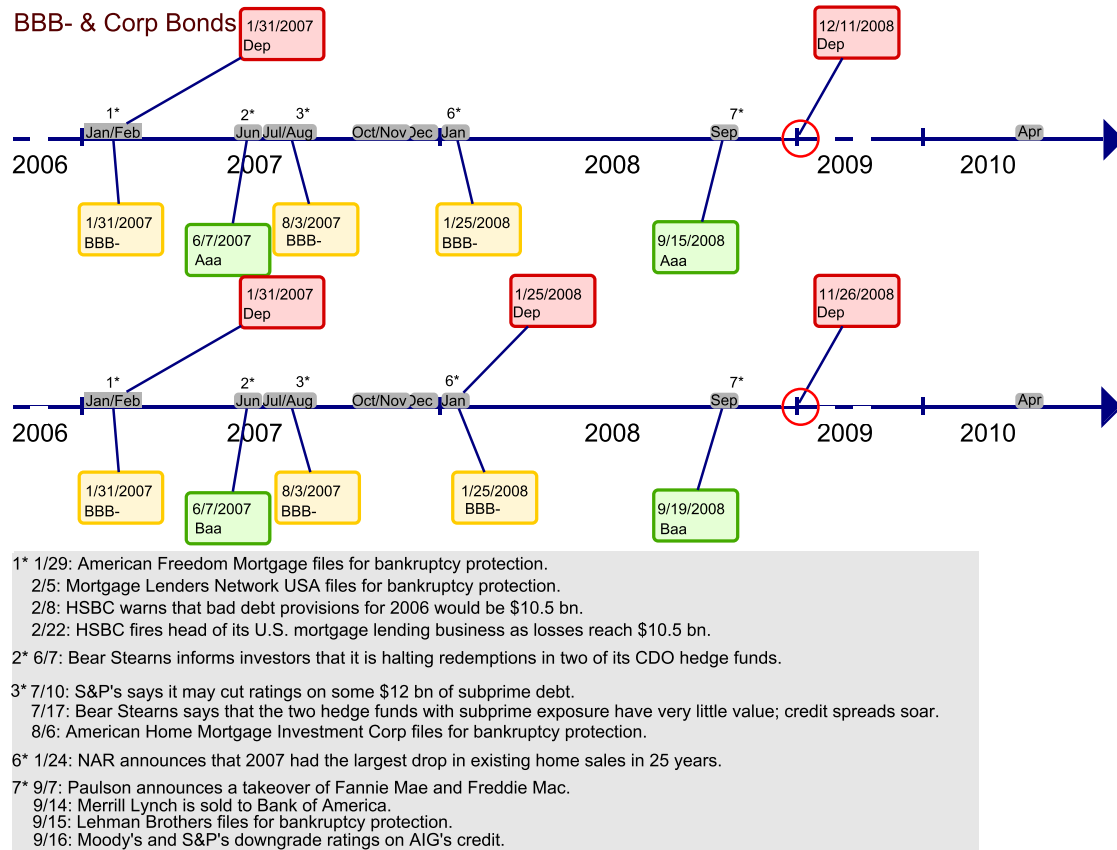


Figure A.13: Detected Abrupt Changes Between ABX\_BBB- and Corporate Bonds

This figure shows the time lines of the abrupt changes between the BBB- index and the corporate bonds (Moody's Aaa and Baa) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the BBB- and the corporate bonds. The green frames indicate the dates when the time series of the corporate bonds change abruptly. The yellow frames correspond to abrupt changes in the BBB-.



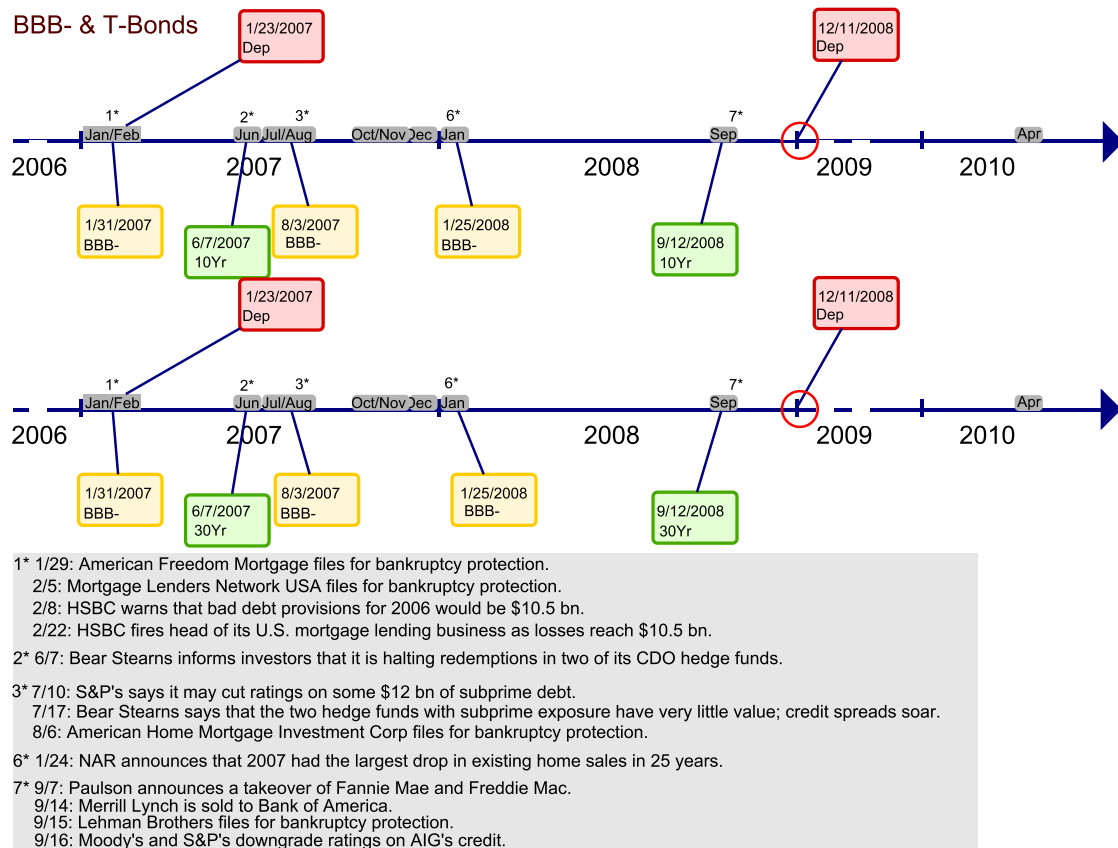
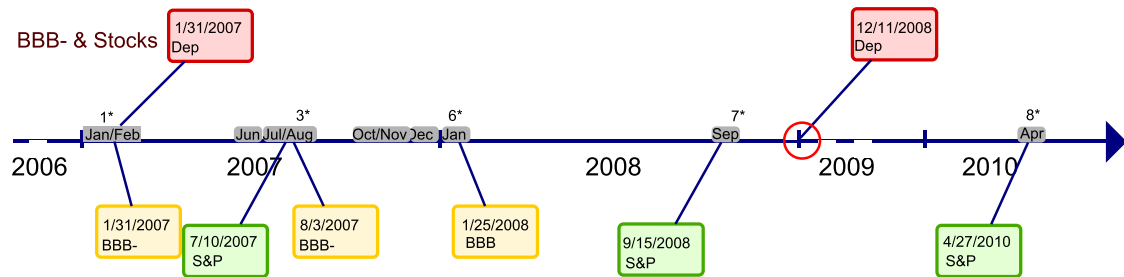


Figure A.14: Detected Abrupt Changes Between ABX\_BBB- and Treasury Bonds

This figure shows the time lines of the abrupt changes between the BBB- index and the Treasury bonds (10 year's and 30 years) and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the BBB- and the Treasury bonds. The green frames indicate the dates when the time series of the Treasury bonds change abruptly. The yellow frames correspond to abrupt changes in the BBB-.



- 1\* 1/29: American Freedom Mortgage files for bankruptcy protection.
- 2/5: Mortgage Lenders Network USA files for bankruptcy protection.
- 2/8: HSBC warns that bad debt provisions for 2006 would be \$10.5 bn.
- 2/22: HSBC fires head of its U.S. mortgage lending business as losses reach \$10.5 bn.
- 3\* 7/10: S&P's says it may cut ratings on some \$12 bn of subprime debt.
- 7/17: Bear Stearns says that the two hedge funds with subprime exposure have very little value; credit spreads soar.
- 8/6: American Home Mortgage Investment Corp files for bankruptcy protection.
- 6\* 1/24: NAR announces that 2007 had the largest drop in existing home sales in 25 years.
- 7\* 9/7: Paulson announces a takeover of Fannie Mae and Freddie Mac.
- 9/14: Merrill Lynch is sold to Bank of America.
- 9/15: Lehman Brothers files for bankruptcy protection.
- 9/16: Moody's and S&P's downgrade ratings on AIG's credit.
- 8\* 4/27: Goldman Sachs defends itself against allegations that it profited from the housing market.
- 4/27: S&P's cuts Greece and Portugal's debt ratings.

Figure A.15: Detected Abrupt Changes Between ABX\_BBB- and S&P 500

This figure shows the time line of the abrupt changes between the BBB- index and the S&P 500 Index and the corresponding possible-influent events. The red rectangular frames indicate the abrupt changes detected in the cross covariance structure between the BBB- and the S&P index. The green frames indicate the dates when the time series of the S&P index changes abruptly. The yellow frames correspond to abrupt changes in the BBB-.

Table A.1: Possible Influential Events

1	1/29/2007	American Freedom Mortgage, Inc. files for Chapter 7 protection.
	2/5/2007	Mortgage Lenders Network USA Inc. files for Chapter 11.
	2/8/2007	HSBC warns that bad debt provisions for 2006 would be 20% higher than expected to roughly \$10.5bn.
	2/22/2007	HSBC fires head of its U.S. mortgage lending business as losses reach \$10.5bn.
2	6/7/2007	Bear Stearns & Co informs investors that it is halting redemptions in two of its CDO hedge funds.
3	7/10/2007	Standard & Poor's says it may cut ratings on some \$12 billion of subprime debt.
	7/17/2007	Bear Stearns says that the two hedge funds with subprime exposure have very little value; credit spreads soar.
	8/6/2007	American Home Mortgage Investment Corporation files for Chapter 11 bankruptcy protection.
4	10/24/2007	Merrill Lynch announces a US\$8.4 billion loss, a sum that credit rating firm Standard & Poor's called "startling".
	11/4/2007	Citigroup announces a further \$8-11 billion of subprime-related writedowns and losses.
5	12/12/2007	Central banks coordinate the launch of the temporary Term Auction Facility (TAF).
6	1/24/2008	The National Association of Realtors (NAR) announces that 2007 had the largest drop in existing home sales in 25 years.
7	9/7/2008	Treasury Secretary Henry Paulson announces a takeover of Fannie Mae and Freddie Mac.
	9/14/2008	Merrill Lynch is sold to Bank of America amidst fears of a liquidity crisis and Lehman Brothers' collapse.
	9/15/2008	Lehman Brothers files for bankruptcy protection.
	9/16/2008	Moody's and Standard and Poor's downgrade ratings on AIG's credit.
8	4/27/2010	Goldman Sachs defends itself on Capitol Hill against allegations that it profited from the housing market collapse.
	4/27/2010	Standard & Poor's cuts Greece and Portugal's debt ratings.

Note: This table reports the influential events around the estimated abrupt changes that are likely the causes of the changes.

## A.2 Mathematical Proofs

**Proof of Theorem 1.** I have

$$f_{T_k}^X(u, \omega) := \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \exp(-i\omega s) \int_{-\pi}^{\pi} \exp(i\lambda s) A_{luT, T_k}^{0*}(\lambda) A_{l'uT-s, T_k}^0(\lambda) d\lambda$$

and

$$f_k^X(u, \omega) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \exp(-i\omega s) \int_{-\pi}^{\pi} \exp(i\lambda s) A_{lk}^*(u, \lambda) A_{lk}(u, \lambda) d\lambda.$$

Therefore for each  $k = 1, \dots, m+1$ ,

$$\begin{aligned} & \int_{-\pi}^{\pi} |f_{T_k}^X(u, \omega) - f_k^X(u, \omega)|^2 d\omega \\ &= \int_{-\pi}^{\pi} \left| \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \exp(-i\omega s) \int_{-\pi}^{\pi} \exp(i\lambda s) (A_{luT, T_k}^{0*}(\lambda) A_{l'uT-s, T_k}^0(\lambda) - A_{lk}^*(u, \lambda) A_{lk}(u, \lambda)) d\lambda \right|^2 d\omega \\ &= \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} |c_s|^2 \end{aligned}$$

where  $c_s = \int_{-\pi}^{\pi} \exp(i\lambda s) g_s(u, \lambda) d\lambda$  with  $g_s(u, \lambda) = A_{luT, T_k}^{0*}(\lambda) A_{l'uT-s, T_k}^0(\lambda) - A_{lk}^*(u, \lambda) A_{lk}(u, \lambda)$ .

By a standard argument for Fourier coefficients (Bary (1964), Chapter 2.3),  $|c_s| \leq C s^{-\alpha}$  and thus

$$\sum_{s=n}^{\infty} |c_s|^2 = O(n^{-2\alpha+1}).$$

On the other hand, via summation by parts,

$$\begin{aligned} \sum_{s=0}^{n-1} |c_s|^2 &= \sum_{s=0}^{n-1} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \exp(i(\mu - \lambda)s) g_s(u, \lambda) g_s^*(u, \mu) d\lambda d\mu \\ &\leq \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| g_{n-1}(u, \lambda) g_{n-1}^*(u, \mu) \sum_{s=0}^{n-1} \exp(i(\mu - \lambda)s) \right. \\ &\quad \left. - \sum_{s=0}^{n-1} [g_s(u, \lambda) g_s^*(u, \mu) - g_{s-1}(u, \lambda) g_{s-1}^*(u, \mu)] \sum_{r=0}^{s-1} \exp(i(\mu - \lambda)r) \right| d\lambda d\mu \\ &\leq \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| g_{n-1}(u, \lambda) g_{n-1}^*(u, \mu) \sum_{s=0}^{n-1} \exp(i(\mu - \lambda)s) \right| d\lambda d\mu \\ &\quad + \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sum_{s=0}^{n-1} \left| [g_s(u, \lambda) g_s^*(u, \mu) - g_{s-1}(u, \lambda) g_{s-1}^*(u, \mu)] \sum_{r=0}^{s-1} \exp(i(\mu - \lambda)r) \right| d\lambda d\mu. \end{aligned}$$

Denote  $\zeta_l(u, \omega) = A_{luT, T_k}^0(\omega) - A_{lk}(u, \omega)$  and  $\eta_{ls}(u, \omega) = A_{lk}(u - \frac{s}{T}, \omega) - A_{lk}(u, \omega)$ . There exist constants  $K_1$  and  $K_2$ , such that

$$\begin{aligned} \sup_{u, \omega} |\zeta_l(u, \omega)| &\leq \frac{K_1}{T_k^2} \\ \sup_{u, \omega} |\eta_{ls}(u, \omega)| &\leq K_2 \left( \frac{s}{T_k} \right)^\alpha. \end{aligned}$$

Therefore,

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| g_{n-1}(u, \lambda) g_{n-1}^*(u, \mu) \sum_{s=0}^{n-1} \exp(i(\mu - \lambda)s) \right| d\lambda d\mu = O\left(\frac{n}{T_k^4}\right)$$

and

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sum_{s=0}^{n-1} \left| \left[ g_s(u, \lambda) g_s^*(u, \mu) - g_{s-1}(u, \lambda) g_{s-1}^*(u, \mu) \right] \sum_{r=0}^{s-1} \exp(i(\mu - \lambda)r) \right| d\lambda d\mu = O\left(\frac{n-1}{T_k^4}\right).$$

As a conclusion,  $\sum_{s=0}^{n-1} |c_s|^2 = O\left(\frac{n}{T_k^4}\right)$  with  $n < T_k^{1-1/\alpha} + 1$ . The same holds for  $\sum_{s=0}^{-\infty} |c_s|^2$ . ■

**Lemma 7** *Asymptotic Mean*

$$E(\zeta(z)) = 2z G_{Re}(\lambda) G'_{Re}(\lambda),$$

$$\text{where } \widehat{G}_{Re}(u) = \widehat{F}_m^+(u) - \widehat{F}_m^-\left(u - \frac{1}{T}\right).$$

**Proof of Lemma 7.** I define

$$\begin{aligned} \zeta_n(z) &= T/m \left( D_m\left(\frac{m}{T}z\right) - D_m(0) \right) \\ &= T/m \left[ \left| \widehat{F}_m^+\left(\lambda + \frac{m}{T}z\right) - \widehat{F}_m^-\left(\lambda + \frac{m}{T}z - \frac{1}{T}\right) \right|^2 - \left| \widehat{F}_m^+(\lambda) - \widehat{F}_m^-\left(\lambda - \frac{1}{T}\right) \right|^2 \right]. \end{aligned}$$

Since

$$\begin{aligned} &\widehat{F}_m^+\left(\lambda + \frac{m}{T}z\right) - \widehat{F}_m^-\left(\lambda + \frac{m}{T}z - \frac{1}{T}\right) \\ &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} K\left(\frac{\omega - \mu}{b}\right) \left[ \widehat{I}_m^+(u, \omega) - \widehat{I}_m^-\left(u - \frac{1}{T}, \omega\right) \right] d\mu d\omega, \end{aligned}$$

where

$$\begin{aligned} &\widehat{I}_m^+(u, \omega) - \widehat{I}_m^-\left(u - \frac{1}{T}, \omega\right) \\ &= \sum_{|s|=0}^{m-1} \widehat{\psi}_s(u) \cos(\omega s) \end{aligned}$$

with

$$\begin{aligned}\widehat{\psi}_s(u) &= \begin{cases} \frac{1}{2\pi H_n} \sum_{t=|s|}^{m-1} h_{t,m} h_{t-|s|,m} \left( X_{lt+uT-m}^+ X_{l't-s+uT-m}^+ - X_{lt+uT-m}^- X_{l't-s+uT-m}^- \right) & \text{when } s \geq 0 \\ \frac{1}{2\pi H_n} \sum_{t=|s|}^{m-1} h_{t,m} h_{t-|s|,m} \left( X_{lt+uT-m}^+ X_{l't+s+uT-m}^+ - X_{lt+uT-m}^- X_{l't+s+uT-m}^- \right) & \text{when } s < 0 \end{cases} \\ &: = \frac{1}{2\pi H_n} \sum_{t=|s|}^{m-1} h_t h_{t-s} Y_{t,s}(u),\end{aligned}$$

I have

$$\begin{aligned}\widehat{G}_{Re}(u) &= \sum_{|s|=0}^{m-1} \widehat{\psi}_s(u) \int_{-\pi}^{v^*} \int_{-\pi}^{\pi} K\left(\frac{\omega-\mu}{b}\right) \cos(\mu s) d\mu d\omega \\ &: = \sum_{|s|=0}^{m-1} \widehat{\psi}_s(u) \phi(s)\end{aligned}$$

So

$$\begin{aligned}&E(\zeta_n(z)) \\ &= T/m \left[ E \left| \widehat{F}_m^+ \left( \lambda + \frac{m}{T} z \right) - \widehat{F}_m^- \left( \lambda + \frac{m}{T} z - \frac{1}{T} \right) \right|^2 - E \left| \widehat{F}_m^+ (\lambda) - \widehat{F}_m^- \left( \lambda - \frac{1}{T} \right) \right|^2 \right] \\ &= (T/m) E \left[ \widehat{G}_{Re} \left( \lambda + \frac{m}{T} z \right)^2 - \widehat{G}_{Re} (\lambda)^2 \right] \\ &= T/m \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) E \left( \widehat{\psi}_s \left( \lambda + \frac{m}{T} z \right) \widehat{\psi}_\tau \left( \lambda + \frac{m}{T} z \right) - \widehat{\psi}_s (\lambda) \widehat{\psi}_\tau (\lambda) \right) \\ &= T/m \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \\ &\quad \frac{1}{4\pi^2 (H_n)^2} \sum_{t_1=|s|}^{m-1} \sum_{t_2=|\tau|}^{m-1} h_{t_1} h_{t_1-s} h_{t_2} h_{t_2-\tau} \left[ E Y_{t_1,s} \left( \lambda + \frac{m}{T} z \right) Y_{t_2,\tau} \left( \lambda + \frac{m}{T} z \right) - E Y_{t_1,s} (\lambda) Y_{t_2,\tau} (\lambda) \right] \\ &= \frac{T}{m} \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \\ &\quad \frac{1}{4\pi^2 (H_n)^2} \sum_{t_1=|s|}^{m-1} \sum_{t_2=|\tau|}^{m-1} h_{t_1} h_{t_1-s} h_{t_2} h'_{t_2-\tau} \left( \delta'_{t_1,t_2,s} (\lambda) \frac{m}{T} z + O\left(\frac{m}{T} z\right)^2 \right) \\ &= \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \frac{1}{4\pi^2 (H_n)^2} \sum_{t_1=|s|}^{m-1} \sum_{t_2=|\tau|}^{m-1} h_{t_1} h_{t_1-s} h_{t_2} h'_{t_2-\tau} \left( \delta'_{t_1,t_2,s} (\lambda) z + \frac{m}{T} O(z)^2 \right)\end{aligned}$$

by Taylor expansion, where I assume that  $\delta_{t_1,t_2,s}(u) = E Y_{t_1,s}(u) Y_{t_2,\tau}(u)$  is a continuous function of  $u$  and is infinitely differentiable in a neighborhood of  $\lambda$ .

I let

$$K_1 = \sum_{|s|=0}^{\infty} \sum_{|\tau|=0}^{\infty} \phi_{Re}(s) \phi_{Re}(\tau) \\ \frac{1}{4\pi^2 (H_n)^2} \sum_{t_1=|s|}^{\infty} \sum_{t_2=|\tau|}^{\infty} h_{t_1} h_{t_1-s} h_{t_2} h'_{t_2-\tau} \delta'_{t_1, t_2, s}(\lambda)$$

such that  $E\zeta_n(z) - K_1 z \rightarrow 0$  as a consequence of dominated convergence. ■

**Lemma 8** *Asymptotic Covariance*

$$Var\left(\sqrt{\frac{m}{T}}\zeta_n(z)\right) = \sum_{|s|=0}^{\infty} \sum_{|\tau|=0}^{\infty} \sum_{|s'|=0}^{\infty} \sum_{|\tau'|=0}^{\infty} \phi_{Re}(s) \phi_{Re}(\tau) \\ \phi_{Re}(s') \phi_{Re}(\tau') (g'_1(\lambda) - 2g'_2(\lambda)),$$

where

$$g_1(u) = Cov(\widehat{\psi}_s(u) \widehat{\psi}_{\tau}(u), \widehat{\psi}_{s'}(u) \widehat{\psi}_{\tau'}(u))$$

and

$$g_2(u) = Cov(\widehat{\psi}_s(u) \widehat{\psi}_{\tau}(u), \widehat{\psi}_{s'}(\lambda) \widehat{\psi}_{\tau'}(\lambda)).$$

**Proof of Lemma 8.**

$$Var(\zeta_n(z)) = E\zeta_n(z)^2 - (E\zeta_n(z))^2$$

with

$$E\zeta_n(z)^2 = \left(\frac{T}{m}\right)^2 E \left[ \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \right. \\ \left. \left( \widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right) \widehat{\psi}_{\tau}\left(\lambda + \frac{m}{T}z\right) - \widehat{\psi}_s(\lambda) \widehat{\psi}_{\tau}(\lambda) \right) \right]^2 \\ = \left(\frac{T}{m}\right)^2 \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \sum_{|s'|=0}^{m-1} \sum_{|\tau'|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \phi_{Re}(s') \phi_{Re}(\tau') \\ E \left( \widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right) \widehat{\psi}_{\tau}\left(\lambda + \frac{m}{T}z\right) - \widehat{\psi}_s(\lambda) \widehat{\psi}_{\tau}(\lambda) \right) \left( \widehat{\psi}_{s'}\left(\lambda + \frac{m}{T}z\right) \widehat{\psi}_{\tau'}\left(\lambda + \frac{m}{T}z\right) - \widehat{\psi}_{s'}(\lambda) \widehat{\psi}_{\tau'}(\lambda) \right)$$

Therefore,

$$\begin{aligned}
\text{Var}(\zeta_n(z)) &= E\zeta_n(z)^2 - (E\zeta_n(z))^2 \\
&= \left(\frac{T}{m}\right)^2 \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \sum_{|s'|=0}^{m-1} \sum_{|\tau'|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \phi_{Re}(s') \phi_{Re}(\tau') \\
&\quad \text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right) - \widehat{\psi}_s(\lambda)\widehat{\psi}_\tau(\lambda), \widehat{\psi}_{s'}\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_{\tau'}\left(\lambda + \frac{m}{T}z\right) - \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) \\
&= \left(\frac{T}{m}\right)^2 \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \sum_{|s'|=0}^{m-1} \sum_{|\tau'|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \phi_{Re}(s') \phi_{Re}(\tau') \\
&\quad \text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right), \widehat{\psi}_{s'}\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_{\tau'}\left(\lambda + \frac{m}{T}z\right)\right) \\
&\quad - 2\text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) + \text{Cov}\left(\widehat{\psi}_s(\lambda)\widehat{\psi}_\tau(\lambda), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) \\
&= \left(\frac{T}{m}\right)^2 \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} \sum_{|s'|=0}^{m-1} \sum_{|\tau'|=0}^{m-1} \phi_{Re}(s) \phi_{Re}(\tau) \phi_{Re}(s') \phi_{Re}(\tau') \\
&\quad \text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right), \widehat{\psi}_{s'}\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_{\tau'}\left(\lambda + \frac{m}{T}z\right)\right) \\
&\quad - \text{Cov}\left(\widehat{\psi}_s(\lambda)\widehat{\psi}_\tau(\lambda), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) \\
&\quad - 2\left(\text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) - \text{Cov}\left(\widehat{\psi}_s(\lambda)\widehat{\psi}_\tau(\lambda), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right)\right) \\
&= \frac{T}{m}K_2z
\end{aligned}$$

The last equation is derived as follows: First I define

$$g_1(u) = \text{Cov}\left(\widehat{\psi}_s(u)\widehat{\psi}_\tau(u), \widehat{\psi}_{s'}(u)\widehat{\psi}_{\tau'}(u)\right)$$

and

$$g_2(u) = \text{Cov}\left(\widehat{\psi}_s(u)\widehat{\psi}_\tau(u), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right),$$

and assume that they are both continuous on  $u$  and infinitely differentiable in a neighborhood of  $\lambda$ . Then

$$\begin{aligned}
&\text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right), \widehat{\psi}_{s'}\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_{\tau'}\left(\lambda + \frac{m}{T}z\right)\right) - \text{Cov}\left(\widehat{\psi}_s(\lambda)\widehat{\psi}_\tau(\lambda), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) \\
&= g_1\left(\lambda + \frac{m}{T}z\right) - g_1(\lambda) = g_1'(\lambda) \frac{m}{T}z
\end{aligned}$$

and

$$\begin{aligned}
&\text{Cov}\left(\widehat{\psi}_s\left(\lambda + \frac{m}{T}z\right)\widehat{\psi}_\tau\left(\lambda + \frac{m}{T}z\right), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) - \text{Cov}\left(\widehat{\psi}_s(\lambda)\widehat{\psi}_\tau(\lambda), \widehat{\psi}_{s'}(\lambda)\widehat{\psi}_{\tau'}(\lambda)\right) \\
&= g_2\left(\lambda + \frac{m}{T}z\right) - g_2(\lambda) = g_2'(\lambda) \frac{m}{T}z.
\end{aligned}$$



Then I let

$$K_2 = \sum_{|s|=0}^{\infty} \sum_{|\tau|=0}^{\infty} \sum_{|s'|=0}^{\infty} \sum_{|\tau'|=0}^{\infty} \phi_{Re}(s) \phi_{Re}(\tau) \phi_{Re}(s') \phi_{Re}(\tau') (g'_1(\lambda) - 2g'_2(\lambda)).$$

■

**Proof of Theorem 2.** I observe

$$\begin{aligned} & \zeta_n(z) - E\zeta_n(z) \\ = & T/m \left[ \left( \left| \widehat{F}_m^+ \left( \lambda + \frac{m}{T}z \right) - \widehat{F}_m^- \left( \lambda + \frac{m}{T}z - \frac{1}{T} \right) \right|^2 - E \left| \widehat{F}_m^+ \left( \lambda + \frac{m}{T}z \right) - \widehat{F}_m^- \left( \lambda + \frac{m}{T}z - \frac{1}{T} \right) \right|^2 \right) \right. \\ & \left. - \left( \left| \widehat{F}_m^+ (\lambda) - \widehat{F}_m^- \left( \lambda - \frac{1}{T} \right) \right|^2 - E \left| \widehat{F}_m^+ (\lambda) - \widehat{F}_m^- \left( \lambda - \frac{1}{T} \right) \right|^2 \right) \right] \\ = & T/m \left[ \left( \widehat{G}_{RE} \left( \lambda + \frac{m}{T}z \right)^2 - E\widehat{G}_{RE} \left( \lambda + \frac{m}{T}z \right)^2 + \widehat{G}_{IM} \left( \lambda + \frac{m}{T}z \right)^2 - E\widehat{G}_{IM} \left( \lambda + \frac{m}{T}z \right)^2 \right) \right. \\ & \left. - \left( \widehat{G}_{RE} (\lambda)^2 - E\widehat{G}_{RE} (\lambda)^2 + \widehat{G}_{IM} (\lambda)^2 - E\widehat{G}_{IM} (\lambda)^2 \right) \right] \\ = & T/m \sum_{|s|=0}^{m-1} \sum_{|\tau|=0}^{m-1} (\phi_{Re}(s) \phi_{Re}(\tau) + \phi_{Im}(s) \phi_{Im}(\tau)) \\ & \left[ \left( \psi_s \left( \lambda + \frac{m}{T}z \right) \psi_\tau \left( \lambda + \frac{m}{T}z \right) - \psi_s(\lambda) \psi_\tau(\lambda) \right) - E \left( \psi_s \left( \lambda + \frac{m}{T}z \right) \psi_\tau \left( \lambda + \frac{m}{T}z \right) - \psi_s(\lambda) \psi_\tau(\lambda) \right) \right] \\ : & = \sum_{i=1}^n W_{(i)} \epsilon_i \end{aligned}$$

where  $i$  is defined as  $(s, \tau)$ ,  $n = (2m-1)^2$ , i.e.,  $(1) = (1-m, 1-m)$ ,  $(2) = (1-m, 2-m)$ , ...,  $(n) = (2m-1, 2m-1)$ .  $\epsilon_i$  is independent with mean 0 and variance 1.

And,

$$\max_{1 \leq i \leq n} \frac{|W_i|}{\sqrt{\sum_i W_i^2}} \rightarrow 0 \text{ as } m \rightarrow \infty.$$

by Lindeberg Central Limit Theorem,

$$\sqrt{\frac{m}{T}} (\zeta_n(z) - K_1 z) \xrightarrow{d} N(0, K_2 z).$$

■

**Proof of Theorem 3.** As in the procedure of showing the asymptotic distribution of  $\widehat{\lambda}$ ,  $\widehat{\lambda}$  is constructed by

$$\widehat{\lambda} = \lambda + \frac{m}{T} z_n,$$

where  $z_n$  is the maximizer of the process  $\zeta_n(z)$  with  $z \in [-M, M]$  for some positive  $M$ .

Therefore,

$$\begin{aligned} & P\left(\left|\widehat{\lambda} - \lambda\right| > \eta\right) \\ &= P\left(\frac{m}{T} |z_n| > \eta\right) \\ &= P\left(|z_n| > \eta \frac{T}{m}\right) \rightarrow 0 \end{aligned}$$

since  $\frac{T}{m} \rightarrow \infty$  as  $T \rightarrow \infty$  and  $|z_n| \leq M$ . ■

## APPENDIX B

### APPENDIX OF CHAPTER 4

**Lemma 9**  *$D_n^2$  is bounded as a function of  $\theta$ .*

**Proof of Lemma 9.** The conclusion can be easily proved by Assumptions 1-2 and 8, (4.8), (4.13), boundedness of the complex exponential function, and Theorem 4.27 of Apostol(1974). ■

**Proof of Theorem 5.** By the Boundedness of  $D_n^2$  in Lemma A.1 which is stronger than Assumption A4 in Andrews (1987) and Assumptions 1, 3 and 7 together, we can invoke Andrews'(1987) main theorem to get:

$$\sup_{\beta} |D_n^2 - D^2| \rightarrow 0 \text{ almost surely(a.s.)}$$

Under Assumption 4.5,  $D^2$  has a unique minimum at  $\theta_0$ . Then further by Theorem 2.1 of Newey and McFadden(1994) or Theorem 2:2 in Domowitz and White(1982), the consistency is obtained. ■

**Proof of Theorem 6.** By (4.13) and Assumptions 4.2, 4.6, 4.7, and 4.8,  $\frac{\partial^2}{\partial \theta^2} D_n^2$  exists and is continuous. From the proof of Theorem 5,

$$D_n^2 \rightarrow D^2 \text{ uniformly in } \theta$$

Further by the continuous mapping theorem and twice continuously differentiability of  $D_n^2$  implied by (4.13) and Assumptions 4.6, 4.8 and 4.10, we have

$$\frac{\partial^2}{\partial \theta^2} D_n^2 \rightarrow^p \frac{\partial^2}{\partial \theta^2} D^2 \text{ uniformly in } \theta \quad (\text{A.1})$$

Following Theorem 3.1 of Newey and McFadden(1994) or Theorem 4.1.3 in Amemiya (1985), we take derivatives of  $D_n^2$  with respect to  $\theta$  and then the first

order condition for (4.13) is obtained:

$$\frac{\partial}{\partial \theta} D_n^2(\theta) \Big|_{\theta=\widehat{\theta}} = 0 \quad (\text{A.2})$$

Further by applying the mean value theorem to  $\frac{\partial}{\partial \theta} D_n^2$  in (A.2),

$$\frac{\partial}{\partial \theta} D_n^2(\theta) \Big|_{\theta=\theta_0} + \frac{\partial^2}{\partial \theta^2} D_n^2(\theta) \Big|_{\theta=\bar{\theta}} (\widehat{\theta} - \theta_0) = 0$$

which gives us

$$\sqrt{n}(\widehat{\theta} - \theta_0) = \left[ \frac{\partial^2}{\partial \theta^2} D_n^2(\theta) \Big|_{\theta=\bar{\theta}} \right]^{-1} \left[ \sqrt{n} \frac{\partial}{\partial \theta} D_n^2(\theta) \Big|_{\theta=\theta_0} \right] \quad (\text{A.3})$$

By Assumption 4.3 and 4.9, we can apply a CLT for mixing sequences to obtain

$$\sqrt{n} \frac{\partial}{\partial \theta} D_n^2|_{\theta=\theta_0} \rightarrow^d N(0, \Sigma) \quad (\text{A.4})$$

where  $\Sigma$  is defined in Assumption A.9. By (A.1), (A.3), (A.4) and Slutsky theorem, the desired conclusion is proved. ■

## BIBLIOGRAPHY

- [1] Acharya, V., Pedersen, L., (2005), "Asset pricing with liquidity risk", *Journal of Financial Economics*, 77, 375-410.
- [2] Allen, F., Gale, D., (2000), 'Financial contagion', *Journal of Political Economy*, 108, 1-33.
- [3] Alvarez, F., Jermann U. J., (2005), "Using asset prices to measure the persistence of the marginal utility of wealth", *Econometrica*, 73(6), 1977-2016.
- [4] Amaro de M., J. and Fernandesbes M., (2007), "Testing the Markov property with high frequency data," *Journal of Econometrics*, 141, 44-64.
- [5] Amemiya, T., (1974), "The nonlinear two-stage least squares estimator," *Journal of Econometrics*, 2, 105-110.
- [6] Amemiya, T., (1977), "The maximum likelihood and nonlinear three-stage least squares estimator in the general nonlinear simultaneous equations model," *Econometrica*, 45, 955-968.
- [7] Amemiya, T., (1985), "Advanced econometrics," Oxford, Blackwell.
- [8] Amihud, Y., (2002), "Illiquidity and stock returns: cross-section and time-series effects", *Journal of Financial Markets*, 5, 31-56.
- [9] Andrews, D. W. K., (1993), "Tests for parameter instability and structural change with unknown change point", *Econometrica*, 61, 821-856 (Corrigendum, 71, 395-397).
- [10] Andrews, D. W. K., and Ploberger, W., (1994), "Optimal tests when a nuisance parameter is present only under the alternative," *Econometrica*, 62, 1383-1414.
- [11] Andrews, D. W. K., Lee, I., and Ploberger W., (1996), "Optimal change point tests for normal linear regression", *Journal of Econometrics*, 70, 9-38.
- [12] Andrews, D. W. K., (1987), "Consistency in nonlinear econometric models: a generic uniform law of large numbers," *Econometrica*, 6, 1465-1471.
- [13] Apostol, T. M., (1974), "Mathematical analysis," Addison-Wesley, Reading, MA.

- [14] Bae, K., Karolyi, G. A., and Stulz R. M., (2003), "A new approach to measuring financial contagion", *Review of Financial Studies*, 16(3), 717-763.
- [15] Bai, J. and Perron P., (1998), "Estimating and testing linear models with multiple structural changes", 66, 47-78.
- [16] Bansal, R. and Yaron A., (2004), "Risks for the long Run: a potential solution of asset pricing puzzles," *Journal of Finance*, 59, 1481–1509.
- [17] Bekaert, G., Harvey, C. R., and Ng, A., (2005), "Market integration and contagion", *Journal of Business*, 78(1), 39-69.
- [18] Berkowitz, J., (2001), "Generalized spectral estimation of the consumption-based asset pricing model," *Journal of Econometrics*, 104, 269-288.
- [19] Beveridge, S., and Nelson, C. R., (1981), "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the "business cycle,"; *Journal of Monetary Economics* 7(2): 151—74.
- [20] Bierens, H. J., (1982), "Consistent model specification tests," *Journal of Econometrics*, 20, 105-134.
- [21] Billio, M., Getmansky M., Lo A. D., and Pelizzon L., (2012), "Econometric measures of connectedness and systemic risk in the finance and insurance sectors", *Journal of Financial Economics*, 104(3), 535-559.
- [22] Billingsley, P. (1995), "Probability and measure", Wiley and Sons.
- [23] Breeden, D.T., (1979), "An intertemporal asset pricing model with stochastic consumption and investment opportunities", *Journal of Financial Economics*, 7(3), 265–296.
- [24] Brown, R.L., Durbin, J., and Evans, J.M., (1975), "Techniques for testing the constancy of regression relationships over time", *Journal of the Royal Statistical Society, B* 37, 149-163.
- [25] Brunnermeier, M. K., (2009), "Deciphering the liquidity and credit crunch 2007-2008", *Journal of Economic Perspectives*, 23(1), 77-100.
- [26] Brunnermeier, M., Pedersen, L., (2009), "Market liquidity and funding liquidity", *Review of Financial Studies*, 22, 2201-2238.

- [27] Burnside, C., (1994), "Hansen–Jagannathan bounds as classical tests of asset pricing models", *Journal of Business and Economic Statistics*, 12, 57–59.
- [28] Campbell, J.Y., Lo, A. W., and MacKinlay, A.C.,(1997), "The econometrics of financial markets", Princeton University Press, Princeton.
- [29] Campbell, J.Y., and Cochrane, J.H. (1999). "By force of habit: a consumption based explanation of aggregate stock market behavior", *Journal of Political Economy*, 107, 205-251.
- [30] Cecchetti, S., Lam, P., and Mark, N., (1994), "Testing volatility restrictions on intertemporal marginal rates of substitution implied by Euler equations and asset returns", *Journal of Finance*, 49, 123–152.
- [31] Chamberlain, G. and M. Rothschild, (1983), "Arbitrage, factor structure, and mean-variance analysis on large asset markets," *Econometrica*, 51, pages 1281-1304.
- [32] Chen, B. and Hong, Y., (2008), "Testing for the Markov property in time series", working paper.
- [33] Chen, B. and Hong Y., (2012), "Testing for smooth structural changes in time series models via nonparametric regression", *Econometrica*, 80(3), 1157 - 1183.
- [34] Chordia T., Sarkar A., Subrahmanyam A., (2005), "An empirical analysis of stock and bond market liquidity", *Review of Financial Studies*, 18(1), 85-129.
- [35] Chordia T., Sarkar A., Subrahmanyam A., (2011), "Liquidity dynamics and cross-autocorrelations", *Journal of Financial and Quantitative Analysis*, 46(03), 709-736.
- [36] Cochrane, J, (2001), "Asset pricing", Princeton University Press.
- [37] Cochrane, J., Hansen, L., (1992), "Asset pricing explorations for macroeconomics", *NBER Macroeconomics Annual 1992*. MIT Press, Cambridge, MA, 115-165.
- [38] Constantinides, G.M., and Duffie, D. (1996). "Asset pricing with heterogeneous consumers", *Journal of Political Economy*, 104, 219-240.

- [39] Dahlhaus, R., (1985). "A functional limit theorem for tapered empirical spectral functions," *Stoch. Proc. Appl.*, 19, 135-149.
- [40] Dahlhaus, R., (1996a), "On the Kullback-Leibler information divergence of locally stationary processes", *Stoch. Proc. Appl.*, 62, 139-168.
- [41] Dahlhaus, R., (1996b), "Asymptotic statistical inference for nonstationary processes with evolutionary spectra", *Athens Conference on Applied Probability and Time Series 2. Lecture Notes in Statist.*, 115, 145-159. Springer, New York.
- [42] Dauhllhaus R., (1997), "Fitting time series models to nonstationary processes", *The Annals of Statistics*, 25(1), 1-37
- [43] Deo, R. S., (2000), "Spectral tests of the martingale hypothesis under conditional heteroskedasticity", *Journal of Econometrics*, 99, 291-315.
- [44] Dornbusch, R., Park, Y., and Claessens, S., (2000), "Contagion: understanding how it spreads", *The World Bank Research Observer*, 15, 177-197.
- [45] Dwivedi Y. and S Subba Rao, (2011), "A test for second-order stationarity of a time series based on the discrete Fourier transform", *Journal of Time Series Analysis*, 32, 68-91.
- [46] Domínguez, M. A. and Lobato, I. N., (2004), "Consistent estimation of models defined by conditional moment restrictions", *Econometrica*, 72, 1601-1615.
- [47] Domowitz, I. and White, H., (1982), "Misspecified models with dependent observations", *Journal of Econometrics*, 20, 35-58.
- [48] Donald, S. G., Imbens, G. W., and Newey, W. K., (2003), "Empirical likelihood estimation and consistent tests with conditional moment restrictions," *Journal of Econometrics*, 117, 55-93.
- [49] Durlauf, S.N., (1991), "Spectral based testing of the martingale hypothesis," *Journal of Econometrics*, 50, 355-376.
- [50] Easley, D and M. O'hara (1987): "Price, trade size, and information in securities markets," *Journal of Financial Economics*, 19, 69-90.



- [51] Eichler, M., G. Motta and R. von Sachs (2011), "Fitting dynamic factor models to non-stationary time series," *Journal of Econometrics*, 163(1), 51-70.
- [52] Engle, R., (2002), "Dynamic conditional correlation: a simple class of Multivariate generalized autoregressive conditional heteroskedasticity models", *Journal of Business and Economic Statistics*, 20, 339-389.
- [53] Escanciano, J. C. and Velasco, C., (2006), "Generalized spectral tests for the martingale difference hypothesis," *Journal of Econometrics*, 134, 151-185.
- [54] Escanciano, J.C. (2006), "Goodness-of-fit tests for linear and nonlinear time series models", *Journal of the American Statistical Association*, 101, 531-541.
- [55] Farley, J. U., Hinich M. J., and McGuire T. W., (1975), "Some comparisons of tests for a shift in the slopes of a multivariate linear time series model", *Journal of Econometrics*, 3, 297-318.
- [56] Ferson W., and Constantinides, G., (1991), "Habit formation and durability in aggregate consumption: empirical tests," *Journal of Financial Economics*, 29, 199-240.
- [57] Feldhutter, P., (2012), "The same bond at different prices: identifying search frictions and selling pressures", *Review of Financial Studies*, 25, 1155-1206.
- [58] Forbes, K. and Rigobon R., (2001), "Contagion in Latin America: definitions, measurement, and policy implications", *Journal of LACEA Economia*.
- [59] Forbes, K. and R. Rigobon, (2002), "No contagion, only interdependence: measuring stock market comovements", *Journal of Finance*, 57(5), 2223-2261, October.
- [60] Forni, M., Hallin M., Lippi M. and Reichlin L., (2000), "The generalized dynamic-factor Model: identification and estimation," *The Review of Economics and Statistics*, 82(4), 540-554.
- [61] Forni, M., Hallin M., Lippi M. and Reichlin L., (2004), "The generalized dynamic factor model consistency and rates," *Journal of Econometrics*, 119(2), 231-255.
- [62] Forni, M., Hallin M., Lippi M. and Reichlin L., (2005), "The generalized

dynamic factor model: one-sided estimation and forecasting," *Journal of the American Statistical Association*, 100, 830-840.

- [63] Forni, M. and Lippi M., (2001), "The Generalized dynamic factor model: representation theory," *Econometric Theory*, 17(06), 1113-1141.
- [64] Forni, M. and Reichlin, L., (1998), "Let's get real: a factor analytical approach to disaggregated business cycle Dynamics," *Review of Economic Studies*, 65(3), 453-73.
- [65] Frank, N., Gonzlez-Hermosillo B., Hesse, H., (2008), "Transmission of liquidity shocks: evidence from the 2007 subprime crisis", IMF Working paper 08/200.
- [66] Frank N., Hesse, H., (2009), "Financial spillovers to emerging markets during the global financial crisis", IMF Working paper 09/104.
- [67] Fratzscher, M., Duca, M., Straub R., (2012), "A global monetary tsunami?: on the spillovers of US quantitative easing", Issue 9195 of Discussion paper, Centre for Economic Policy Research.
- [68] Geweke, J., (1977), "The dynamic factor analysis of economic time series", in Dennis J. Aigner and Arthur S. Goldberger (Eds.), *Latent Variables in Socio-Economic Models*, North-Holland.
- [69] Hou, K., (2007), "Industry information diffusion and the Lead-lag effect in stock returns," *Review of Financial Studies*, 20(4), 1113-1138.
- [70] Hansen, L. P., (1982), "Large sample properties of generalized method of moments estimators," *Econometrica*, 50, 1029-1054.
- [71] Hansen, L. P., (2007), "Generalized eethods of moments estimation," forthcoming in the Palgrave dictionary.
- [72] Hansen, L. P., J. C. Heaton, N. LI (2004): "Consumption strikes back," Working paper, University of Chicago.
- [73] Hansen, L. P., and Singleton. K. J., (1982), "Generalized instrumental variables of nonlinear rational expectations models," *Econometrica*, 50, 1269-1286

- [74] Hansen, L.P., Singleton, K.J., (1983), "Stochastic consumption, risk aversion, and the temporal behavior of asset returns", *Journal of Political Economy* 91(2), 249–265.
- [75] Hansen, L. P., and Singleton. K. J., (1996), "Efficient estimation of linear asset pricing models with moving average errors," *Journal of Business and Economic Statistics*, 14, 237-274.
- [76] Han, S., and Zhou, H., (2008), "Effects of bond liquidity on the nondefault component of corporate bond spreads: evidence from intraday transactions data", Working paper, Federal Reserve Board.
- [77] Hong, Y., (1999), "Hypothesis testing in time series via the empirical characteristic function", *Journal of the American Statistical Association*, 84, 1201–1220.
- [78] Hong, Y., and Lee Y. J., (2005), "Generalized spectral tests for conditional mean models in time series with conditional heteroscedasticity of unknown form," *Review of Economic Studies*, 72, 499-541.
- [79] Karolyi, G. A., Lee, K.-H., and van Dijk, M., (2012), "Understanding commonality in liquidity around the world", *Journal of Financial Economics*, 105, 83-103.
- [80] King, M., Wadhwani, S., (1990), "Transmission of volatility between stock markets", *Review of Financial Markets*, 3, 5-33.
- [81] Kiyotaki, N., Moore, J., (2002), "Evil is the root of all money", *American Economic Review*, 92, 62-66.
- [82] Kodres, L., Pritsker, M., (2002), "A rational expectations model of financial contagion", *Journal of Finance*, 57, 769-800.
- [83] Kitamura, Y., Tripathi G., and Ahn H., (2004), "Empirical likelihood-based inference in conditional moment restriction models", *Econometrica*, 72, 1167-1714.
- [84] Kyle, A., (1985), "Continuous auctions and insider trading", *Econometrica*, 53, 1315-1335.
- [85] Lee, K.-H., (2011), "The world price of liquidity risk", *Journal of Financial Economics*, 99, 136-161.

- [86] Lin, C.J. and Terasvirta T., (1994);, "Testing the constancy of regression parameters against continuous structural change," *Journal of Econometrics*, 62, 211-228.
- [87] Lo, A. W. and A. C., MacKinlay, (1990), "When are contrarian profits due to stock market overreaction?," *Review of Financial Studies*, Society for Financial Studies, 3(2), 175-205.
- [88] Longin, F., and Solnik B., (1995), "Is the correlation in international equity returns constant: 1960-1990?," *Journal of International Money and Finance*, 14, 3-26.
- [89] Longin, F., and Solnik (2001), "Extreme correlation of international equity markets", *Journal of Finance*, 56, 651-678.
- [90] Longstaff, F. A., (2010), "The subprime credit crisis and contagion in financial markets," *Journal of Financial Economics*, 97(3), 436-450.
- [91] Lucas, R.E., (1978), "Asset prices in an exchange economy", *Econometrica*, 46, 1426-1446.
- [92] Mehra, R., Prescott, E.C., (1985), "The equity premium: a puzzle", *Journal of Monetary Economics*, 15(2), 145-161.
- [93] Newey, W. K. (1990), "Efficient instrumental variables estimation of non-linear models", *Econometrica*, 58, 809-837.
- [94] Newey, W. K. (1993). "Efficient estimation of models with conditional moment restrictions", in G. Maddala, C. Rao, and H. Vinod (eds.), *Handbook of Statistics*, 11, pp. 419-454. Elsevier Science.
- [95] Newey, W. K. and McFadden, D., (1994), "Large sample estimation and hypothesis test", *Handbook of Econometrics*, 4, 2111-2245.
- [96] Newey, W. K. and West, K. D., (1987) "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix", *Econometrica*, 55(3): 703-708
- [97] Paparoditis, E., (2009), "Testing temporal constancy of the spectral structure of a time series," *Bernoulli*, 15(4), 1190-1221.

- [98] Perron, P., (2006), "Dealing with Structural Breaks", in *Palgrave Handbook of Econometrics*, Vol. 1: *Econometric Theory*, K. Patterson and T.C. Mills (eds.), Palgrave Macmillan, 2006, 278-352.
- [99] Picard, D., (1985), "Testing and estimating change-points in time series", *Adv. Appl. Probab.* 17 841–867.
- [100] Priestley, M.B., (1981), "Spectral analysis and time series," Academic Press, San Diego.
- [101] Priestley, M. B., (1965), "Evolutionary spectra and non-stationary processes", *Journal of the Royal Statistical Society. Series B (Methodological)*, 27(2), 204-237
- [102] Qu, Z. and P. Perron, (2007), "Estimating and testing multiple structural changes in multivariate regressions", *Econometrica*, 75, 459-502
- [103] Quandt, R.E., (1960), "Tests of the hypothesis that a linear regression system obeys two separate regimes", *Journal of the American Statistical Association* 55, 324-330.
- [104] Robinson, P. M., (1987), "Asymptotically efficient estimation in the presence of heteroskedasticity of unknown form," *Econometrica*, 55, 875-891.
- [105] Robinson, P. M., (1991), "Best nonlinear three-stage least squares estimation of certain econometric models," *Econometrica*, 59, 755-786.
- [106] Sargent T. J. and C. A.Sims, (1977), "Business cycle modeling without pretending to have too much a priori economic theory," *Working Papers* 55, Federal Reserve Bank of Minneapolis.
- [107] Singleton. K. J., (2006), "Empirical dynamic asset pricing: model specification and econometric assessment," Princeton University Press.
- [108] Song, Z., (2009), "Infinitesimal operator based conditional GMM estimation of diffusion models", *Working paper*, Cornell University.
- [109] Stock, J. H. and M. W., Watson, (2008), "Forecasting in dynamic factor models subject to structural instability", in *The Methodology and Practice of Econometrics, A Festschrift in Honour of Professor David F. Hendry*, Castle, J. and N. Shephard (Eds.), Oxford University Press, Oxford.

- [110] Vayanos, D., (2004), "Flight to quality, flight to liquidity, and the pricing of risk", Working paper, London School of Economics.
- [111] Vogelsang, T. J., (1999), "Sources of nonmonotonic power when testing for a shift in mean of a dynamic time series", *Journal of Econometrics*, 88, 283-299.
- [112] von Sachs, R. and Neumann, M.H., (2000), "A wavelet-based test for stationarity", *J. Time Ser. Anal.* 21 597–613.
- [113] Zheng, J., (2012), "Detection of abrupt structural changes: a spectral approach", Working paper, Cornell University.